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DNS and regularization modeling of a turbulent differentially heated cavity of aspect ratio 5

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ABSTRACT

This work is devoted to the study of turbulent natural convection flows in differentially heated cavities. The adopted configuration corresponds to an air-filled ($Pr = 0.7$) cavity of aspect ratio 5 and Rayleigh number $Ra = 4.5 \times 10^{10}$ (based on the cavity height). Firstly, a complete direct numerical simulation (DNS) has been performed. Then, the DNS results have been used as reference solution to assess the performance of symmetry-preserving regularization as a simulation shortcut: a novel class of regularization that restrain the convective production of small scales of motion in an unconditionally stable manner. In this way, the new set of equations is dynamically less complex than the original Navier–Stokes equations, and therefore more amenable to be numerically solved. Direct comparison with the DNS results shows fairly good agreement even for very coarse grids.

1. Introduction

Natural convection in differentially heated cavities (DHC) has been the subject of numerous studies over the past decades. They model many engineering applications such as ventilation of rooms, cooling of electronic devices or air flow in buildings. Simultaneously, since the pioneering works by De Vahl Davis [1] and Ghia et al. [2], flows in enclosed cavities has served as prototype for the development of numerical algorithms (examples thereof can be found in [3–6], for instance). A schema of the DHC problem is displayed in Fig. 1 (left). An accurate prediction of the flow structure and the heat transfer in such a configuration is of great interest and despite the great effort devoted (see for instance [7–12]) for an accurate turbulence modeling of this configuration it remains a great challenge. This is mainly due to the complex behavior exhibit (see Fig. 1, right): the boundary layers remain laminar in their upstream part up to the point where the waves traveling downstream grow up enough to disrupt the boundary layers ejecting large unsteady eddies. The mixing effect of these eddies results in almost isothermal hot upper and cold lower regions, and forces the temperature drop in the core of the cavity to occur in a smaller region. Therefore, an accurate prediction of the transition point is crucial to determine correctly the flow structure in the cavity. However, the high sensitivity of the thermal boundary layer to external disturbances makes it difficult to predict. In conclusion, the DHC is a challenging configuration for turbulence modeling since areas with completely different regimes coexist and interplay.

1.1. DNS and regularization modeling of turbulence

Here, the adopted configuration corresponds to an air-filled ($Pr = 0.7$) cavity of height aspect ratio $A_3 = 5$ at Rayleigh number $Ra = 4.5 \times 10^{10}$ (based on the cavity height, L_3). This resembles the pioneering experimental set-up performed by Cheesewright et al. [13] in the mid-80s. Since then, their results have been widely used for benchmarking purposes to validate turbulence models (see [14–19,11,12], for instance); therefore, the availability of accurate numerical results is of extreme importance. To that end, a new complete direct numerical simulation (DNS) has been performed. To do so, the incompressible Navier–Stokes (NS) equations have been discretized preserving the (skew) symmetries of the underlying continuous differential operators [20]. In this way, certain fundamental properties such as the inviscid invariants – kinetic energy, enstrophy (in 2D) and helicity (in 3D) – are exactly preserved in a discrete sense. However, DNS at high Ra numbers is not feasible. Therefore, a dynamically less complex mathematical formulation is needed.

In the quest for such a formulation, we consider regularizations [21] (smooth approximations) of the convective term that preserve the symmetry and conservation properties exactly. This require-

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Nomenclature

A_1, A_3	depth and height aspect ratios, L_1/L_2 and L_3/L_2
C	dimensionless stratification, $\partial\langle\theta\rangle/\partial x_3\|_{\left\{x_2=\frac{1}{2A_3}, x_3=1/2\right\}}$
$\mathbf{C}(\mathbf{u}_h)$	discrete convective operator
$\mathbf{D}$	discrete diffusive operator
$\mathbf{f}$	dimensionless body force
$\mathbf{F}$	discrete filter, $\overline{\mathbf{u}_h} = \mathbf{F}\mathbf{u}_h$
g	gravitational acceleration
$\mathbf{L}$	discrete Laplacian operator, $-\mathbf{M}\mathbf{\Omega}^{-1}\mathbf{M}^t$
L_1, L_2, L_3	cavity depth, width and height
L_{ref}	reference length, L_3
$\mathbf{M}$	discrete divergence operator
N	dimensionless Brunt-Väisälä frequency, $(CPr)^{0.5}/(2\pi)$
$Nu(x_3)$	Nusselt number distribution at the hot wall, $-\partial\langle\theta\rangle/\partial x_2\|_{x_2=0}$
Nu	Nusselt number, $\int_0^1 Nu(x_3)dx_3$
$Nu_c(t)$	Nusselt number through the vertical mid-plane, $\int_0^1 (U_2\theta - \partial\theta/\partial x_2)\|_{x_2=\frac{1}{2A_3}} dx_3$
N_1, N_2, N_3	number of nodes in the x_i-direction
p	dimensionless dynamic pressure
p_{ref}	reference dynamic pressure, $\rho(\alpha^2/L_3^2)Ra$
Pr	Prandtl number, ν/α
Ra	Rayleigh number based on cavity height, $(g\beta\Delta T L_3^3)/(\nu\alpha)$
$R_{\phi\phi}(x_1,x_2,r_3)$	two-point correlation, $\langle\phi'(x_1,x_2,x_3)\phi'(x_1,x_2,x_3+r_3)\rangle/\langle(\phi'(x_1,x_2,x_3))^2\rangle$
t	dimensionless time
t_{ref}	reference time, $(L_3^2/\alpha)Ra^{-1/2}$
T	temperature
ΔT	temperature difference, $(T_H - T_C)$
$\mathbf{u}$	dimensionless velocity vector field, $\mathbf{u} = (u_1, u_2, u_3)$
u_{ref}	reference velocity, $(\alpha/L_3)Ra^{1/2}$
(x_1,x_2,x_3)	dimensionless spatial coordinates
x_3^{Tr}	x_3-position of $\sigma(Nu)_{max}$ on the vertical hot wall

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
γ_i	mesh concentration parameters
Δt	time step
Δx_i	mesh size in x_i-directions
ϵ	filter length
θ	dimensionless temperature, $(T-(T_H+T_C)/2)/(T_H-T_C)$
θ_{avg}^{top}	dimensionless averaged temperature at the top wall
μ_{Nu}	first moment of $Nu(x_3)$ about $x_3 = 0.5$, $\int_0^1 (0.5 - x_3) Nu(x_3)dx_3$
ν	kinematic viscosity
ρ	fluid density
$\sigma(\cdot)$	standard deviation
$\boldsymbol{\omega}$	vorticity, $\nabla \times \mathbf{u}$
$\mathbf{\Omega}$	diagonal matrix with sizes of control volumes

Subscripts

C	cold wall
f	index for faces of control volumes
h	discrete scalar or vector field
H	hot wall
max	maximum value
min	minimum value
ref	reference quantity

Superscripts

$(\cdot)'$	fluctuations around the mean value
$\langle\cdot\rangle$	time-averaged
$\widehat{(\cdot)}_k$	Fourier coefficient at wavenumber k
$\overline{(\cdot)}$	linear filter
$(\cdot)^*$	complex conjugate

ment yielded a novel class of regularizations [22] that restrain the convective production of smaller and smaller scales of motion in an unconditionally stable manner. The numerical algorithm used to solve the governing equations preserves the symmetries and conservation properties too [20] and is therefore well-suited to test the proposed simulation model. The regularization makes use of a normalized self-adjoint filter. In the initial tests [22,23], the performance of the method was tested keeping the ratio filter length/grid width constant. Thus, this parameter had to be prescribed in advance and therefore a convergence analysis was needed. Later, to circumvent this, a parameter-free approach was proposed [24]. To do so, we proposed to determine the regularization parameter (the local filter length) dynamically from the requirement that the vortex-stretching must be stopped at the scale set by the grid. However, in this way, some of the basic properties of the filter (*i.e.*, symmetry, normalization, incompressibility ...) are lost. Therefore, they need to be restored by explicitly forcing them. However, such *a posteriori* modifications are artifacts that may change the dynam-

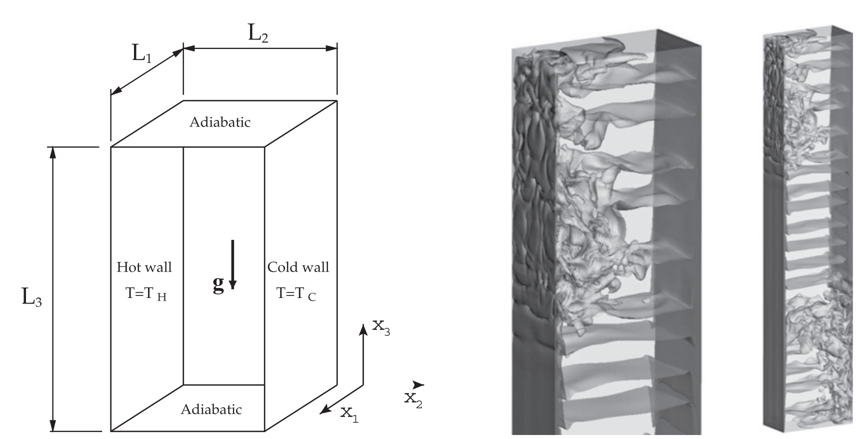

Fig. 1. DHC schema (left) and instantaneous isotherms corresponding to the simulation on MeshA (right).