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Linear and nonlinear double-diffusive convection in a saturated anisotropic porous layer with Soret effect and internal heat source

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ABSTRACT

Double-diffusive convection in an anisotropic porous layer heated and salted from below with an internal heat source and Soret effect is studied analytically using both linear and nonlinear stability analyses. The generalized Darcy model including the time derivative term is employed for the momentum equation. The critical Rayleigh number and wavenumber for stationary and oscillatory modes and frequency of oscillations are obtained analytically using linear theory. The effects of the anisotropy parameters, concentration Rayleigh number, internal heat source, Soret parameter, Vadasz number and Lewis number on the stationary, oscillatory, and heat and mass transfer are shown graphically. Heat and mass transfer have been obtained in terms of the Nusselt number and Sherwood number.

1. Introduction

The problem of double-diffusive convection in porous media has received considerable attention in recent years, on account of its wide applications in, for example, the effect of contaminants on lakes and under ground water, atmospheric pollution, chemical processes, food processing, energy storage, and materials processing. In double diffusion, the buoyancy force is affected not only by the difference of temperature, but also by the difference of concentration of the fluid. A detailed discussion on double diffusion natural convection can be found in the books by Nield and Bejan [1], Ingham and Pop [2] and Vafai [3]. The onset of thermal instability in a horizontal porous layer saturated with Newtonian fluid was first studied by Horton and Rojers [4] and Lapwood [5]. Other studies on linear and nonlinear double diffusive convection in a porous layer were given by Haajizadeh et al. [6], Gaikwad et al. [7], Charrier-Mojtabi et al. [8], Malashetty and Swamy [9] and Capone et al. [10].

Early studies on convection in a porous medium have usually been concerned with homogeneous isotropic porous structures. In the last one decade, the effects of non-homogeneity and anisotropy of the porous medium have been studied. The geological and pedagogical processes rarely form isotropic media as is usually assumed in transport studies. In geothermal system with a ground structure composed of many strata of different permeability, the overall horizontal permeability may be up to 10 times as large as the vertical component. Processes such as sedimentation, compaction, frost action, and reorientation of the solid matrix are responsible for the creation of anisotropic natural porous media. Anisotropy can also be a characteristic of artificial porous material like pelleting used in chemical engineering process, fiber materials used in insulating purposes. Anisotropy finds also application in mathematical modeling in geothermal systems such as fractured rocks [11]. The first study in a fluid layer saturated anisotropic porous medium was conducted by Castinel and Combarnous [12]. Many other studies were conducted in a fluid layer saturated anisotropic porous medium [13,14]. Recently, many authors have studied the effect of anisotropy on the onset of convection in a porous layer [15,9,16,11,17].

A situation in which a porous medium has its own source of heat can occur through radioactive decay or through, in the present perspective, a relatively weak exothermic reaction which can take place within the porous material. Internal heat generation becomes very important in geophysics, reactor safety analysis, metal waste form development, fire and combustion studies and storage of radioactive materials. The onset of convection in a horizontal layer of an anisotropic porous medium with an internal heat source subjected to an inclined temperature gradient was studied by Parthiban and Parthiban [18]. An analytical solution for small Rayleigh number in a finite container with isothermal walls and uniform heat generation within the porous medium was given by

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Nomenclature

d	hight of the porous layer
D	cross diffusion due to T component
Da	Darcy number, K_z/d^2
g	gravitational acceleration
$\mathbf{K}$	permeability of the porous medium, $K_x(\hat{i}\hat{i}+\hat{j}\hat{j})+K_z(\hat{k}\hat{k})$
Le	Lewis number κ_{T_z}/κ_s
Nu	Nusselt number
P	pressure
Pr	Prandtl number
$\mathbf{q}$	velocity vector, (u,v,w)
Ra_T	thermal Rayleigh number, $\left(\frac{\beta_T g \Delta T d K_z}{\nu \kappa_{T_z}}\right)$
Ra_s	concentration Rayleigh number, $\left(\frac{\beta_s g \Delta C d K_z}{\nu \kappa_{T_z}}\right)$
R_i	internal Rayleigh number
Sh	Sherwood number
S_r	Soret parameter, $\left(\frac{D\beta_s}{\kappa_{T_z}\beta_T}\right)$
T	temperature
t	time
Va	Vadasz number, Pr/Da
x,y,z	space coordinates

Greek symbols

α	wavenumber
β_s	concentration expansion coefficient
β_T	thermal expansion coefficient
η	thermal anisotropy parameter
κ_{T_z}	thermal diffusivity
κ_s	concentration diffusivity
μ	dynamic viscosity
ν	kinematic viscosity
ψ	stream function
ρ	density
ξ	mechanical anisotropy parameter

Joshi et al. [19]. Recently, Bhadauria et al. [16] studied the natural convection in a rotating anisotropic porous layer with internal heat generation using a weak nonlinear analysis. In [17], Bhadauria studied double-diffusive natural convection in an anisotropic porous layer in the presence of an internal heat source.

If the cross diffusion terms are included in the species transport equations, then the situation will be quite different. Due to the cross diffusion effect, each property gradient has a significant influence on the flux of the other property. A flux of salt caused by a spatial gradient of temperature is called the Soret effect. Similarly, a flux of heat caused by a spatial gradient of concentration is called the DuFour effect. The DuFour coefficient is of order of magnitude smaller than the Soret coefficient in liquids, and the corresponding contribution to the heat flux may be neglected. Many studies can be found in the literature concerning the Soret and DuFour effects. A study by Rudaraiah and Malashetty [20] discussed the double-diffusive convection in a porous medium in the presence of Soret and DuFour effects. In another study, Rudraiah and Siddheshwar [21] investigated a weak nonlinear stability analysis of double-diffusive convection with cross-diffusion in a fluid saturated porous medium. Bahloul et al. [22] studied analytically and numerically a double-diffusive and Soret-induced convection in a shallow horizontal porous layer. Mansour et al. [23] investigated the multiplicity of solutions induced by thermosolutal convection in a square porous cavity heated from below and subject to horizontal solute gradient in the presence of Soret effect. Recently, Malashetty and Biradar [24] carried out an analytical study of linear and nonlinear double-diffusive convection in a fluid-saturated porous layer with Soret and DuFour effects. Also, a study by Malashetty et al. [25] investigated Soret effect on double diffusive convection in a Darcy porous medium saturated with a couple stress fluid.

The aim of this work is to study the effects of an internal heat source and Soret effect on the convection in a binary fluid-saturated anisotropic porous layer. The onset criteria for stationary and oscillatory double-diffusive convection are obtained using linear stability analysis. Heat and mass transfer have been studied using nonlinear stability analysis, and the results were presented graphically in terms of the Nusselt number Nu and Sherwood number Sh.

2. Mathematical formulation

The physical model under consideration is an infinite horizontal anisotropic porous layer with internal heat source, confined between two parallel horizontal planes at $z = 0$ and $z = d$ with a distance d apart. A Cartesian frame of reference is chosen in such a way that the origin lies on the lower plane and the z-axis as vertically upward, where the gravity force g is acting vertically downward. Adverse temperature and concentration gradients are applied across the porous layer, and the lower and upper planes are kept, respectively, at temperatures $T_0 + \Delta T$ and T_0, and concentrations $S_0 + \Delta S$ and S_0, where ΔT and ΔS are temperature difference and concentration difference between the walls, respectively. The Soret effect is considered and the Oberbeck–Boussinesq approximation is assumed to account for the effect of density variations. Under these assumptions, the governing equations are given by Gaikwad et al. [11]

$$\nabla \cdot \mathbf{q} = 0 \tag{1}$$

$$\frac{\rho_0}{\phi}\frac{\partial \mathbf{q}}{\partial t} = -\nabla P - \frac{\mu}{\mathbf{K}}\mathbf{q} + \rho \mathbf{g} \tag{2}$$

$$\sigma\frac{\partial T}{\partial t} + (\mathbf{q}\cdot\nabla)T = \nabla\cdot(\kappa_{T_z}\cdot\nabla T) + Q(T - T_0) \tag{3}$$

$$\phi\frac{\partial C}{\partial t} + (\mathbf{q}\cdot\nabla)C = (\kappa_s\nabla^2 C) + D\nabla^2 T \tag{4}$$

$$\rho = \rho_0[1 - \beta_T(T - T_0) - \beta_s(C - C_0)] \tag{5}$$

subject to the following boundary conditions

$$T = T_0 + \triangle T, \quad C = C_0 + \triangle C \quad \text{at} \quad z = 0 \tag{6}$$

$$T = T_0, \quad C = C_0 \quad \text{at} \quad z = d \tag{7}$$

where $\mathbf{q}$ is velocity (u,v,w), μ is the dynamic viscosity, Q is internal heat source, $\mathbf{K}$ is the permeability tensor $K_x(\hat{i}\hat{i}+\hat{j}\hat{j})+K_z(\hat{k}\hat{k})$, κ_{T_z} is the thermal diffusivity, T is temperature, β_T is the thermal expansion coefficient, β_s is the concentration expansion coefficient, κ_s is concentration diffusivity of the fluid, σ is the ratio of heat capacities, D is the cross diffusion due to T components, ρ is the density, $\mathbf{g} = (0,0,-g)$ is the gravitational acceleration and ρ_0 is the reference density.

2.1. Basic state

The basic state of the fluid is assumed to be quiescent, and is given by

$$\mathbf{q}_b = (0,0,0), \quad P = P_b(z), \quad T = T_b(z), \quad C = C_b(z), \quad \rho = \rho_b(z) \tag{8}$$