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A study of turbulent heat transfer in curved pipes by numerical simulation

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A B S T R A C T

Turbulent heat transfer in curved pipes was studied by numerical simulation. Two curvatures δ (pipe radius a/curvature radius c) were considered, 0.1 and 0.3; results were also obtained for a straight pipe ($\delta = 0$) for comparison purposes. A tract of pipe 5 diameters in length was chosen as the computational domain and was discretized by finite volume multiblock-structured grids of $\sim 5.3 \times 10^6$ hexahedral cells. Fully developed conditions were assumed; the friction velocity Reynolds number was 500, corresponding to bulk Reynolds numbers between 12 630 and ~17 350 according to the curvature, while the Prandtl number was 0.86 (representative of saturated liquid water at 58 bar). Simulations were protracted for 20 LETOT's a/u_τ; the last 10 LETOT's were used to compute first and second order time statistics, including rms fluctuating temperature and turbulent heat fluxes.

In curved pipes, time averaged results exhibited Dean circulation and a strong velocity and temperature stratification in the radial direction. Turbulence and heat transfer were strongly asymmetric, with higher values near the outer pipe bend. In the outer region, counter-gradient heat transport by turbulent fluxes was observed. For a given friction velocity Reynolds number, overall turbulence levels were lower than in a straight pipe; nevertheless, heat transfer rates were larger due to the curvature-induced modifications of the mean flow and temperature fields.

1. Problem definition

Curved pipes are commonly encountered in heat transfer equipment [1]. A tract of a curved pipe with its axis lying in a plane and a constant curvature is actually a tract of a toroidal pipe; in this case, strictly speaking, the assumption of fully developed flow and thermal fields is compatible only with a closed torus, which, of course, poses realizability problems [2]. However, the associated complications can be avoided in fully turbulent flow: in this case, in fact, the flow and thermal fields are not overly sensitive to the geometry [3] while a small torsion (helical coils) or a small variation of the curvature along the pipe (spiral coils) allow arbitrary pipe lengths to be attained. Therefore, in the present work we will simulate a finite tract of a toroidal pipe while at the same time assuming fully developed conditions.

Fig. 1 shows the computational domain. The radius of curvature is indicated by c and the radius of the cross section by a. The inner and outer bend sides are indicated by "I" and "O", respectively. The azimuthal angle θ is measured in the anti-clockwise direction looking from upstream, with $\theta(O) = 0$, $\theta(I) = \pi$. The coordinates r (radius), θ (local azimuth) and Θ (global azimuth) form an orthogonal reference frame which is the most natural to represent vectors (e.g. velocities and turbulent heat fluxes) and tensors (e.g. Reynolds stresses); the global azimuth Θ can be replaced by the curvilinear abscissa $s = c\Theta$ measured along the axis. However, simulations were run in the Cartesian reference frame (x,y,z), and velocities computed on an arbitrary cross section were then projected onto the (r,θ,s) frame before being processed as discussed in Section 3.4.

In the following, the cross-sectional average of a generic quantity ϕ (e.g. temperature T) will be indicated by ϕ_{av} and its time average by $\bar{\phi}$, while the fluctuation $\phi - \bar{\phi}$ will be indicated by ϕ' and its root mean square value $(\overline{\phi'^2})^{1/2}$ by ϕ_{rms}. The notation $\langle\phi\rangle$ will be used for the wall average of a wall-related quantity ϕ (e.g. the wall heat flux q''_w). The bulk Reynolds number Re will be defined on the basis of the pipe diameter $2a$ and the time- and cross-section-averaged (bulk) velocity $\bar{u}_{av}$ as $\mathrm{Re} = \bar{u}_{av}2a/\nu$, ν being the kinematic viscosity of the fluid. The friction-velocity Reynolds number will be defined as $\mathrm{Re}_\tau = u_\tau a/\nu$, being $u_\tau = (\langle\bar{\tau}_w\rangle/\rho)^{1/2}$ the friction velocity based on the time- and wall-averaged wall shear stress. Global wall scales will be based on u_τ, i.e., u_τ itself for velocity, ν/u_τ for length, u_τ^2 for Reynolds stresses and turbulence energy, etc.; the corresponding normalized quantities will be denoted by a superscript +. The temperature wall scale will be $T_\tau = \langle\bar{q}''_w\rangle/(\rho c_p u_\tau)$, $\langle\bar{q}''_w\rangle$ being the wall-averaged heat flux; in this way, the wall scale for heat fluxes, including the turbulent (Reynolds) heat flux $\rho c_p\overline{u'_iT'}$, will be

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Nomenclature

a	pipe radius [m]
c	curvature radius [m]
c_p	specific heat [J kg^{-1} K^{-1}]
De	Dean number, $\mathrm{Re}\sqrt{\delta}$
f	Darcy friction coefficient
N	number of control volumes
Nu	Nusselt number, $2a\langle q''_w\rangle/[\lambda(T_w - T_b)]$
Pr	Prandtl number
p	pressure [Pa]
$\mathbf{p}_s$	opposite of streamwise pressure gradient [Pa m^{-1}]
q''	heat flux [W m^{-2}]
r	radial coordinate in the cross section [m]
Re	bulk Reynolds number, $\bar{u}_{av}2a/\nu$
Re_τ	friction Reynolds number, $u_\tau a/\nu$
s	curvilinear abscissa along the pipe axis [m]
S	heat source term [K s^{-1}]
T	temperature [K]
T_τ	wall temperature scale, $\langle \bar{q}''_w\rangle/(\rho c_p u_\tau)$ [K]
u_i	i-th velocity component [m s^{-1}]
u_τ	friction velocity, $(\langle\bar{\tau}_w\rangle/\rho)^{1/2}$ [m s^{-1}]
x, y, z	Cartesian coordinates [m]
y^+	distance from the wall in wall units, yu_τ/ν

Greek symbols

δ	dimensionless curvature, a/c
λ	thermal conductivity [W m^{-1} K^{-1}]
ν	kinematic viscosity [m^2 s^{-1}]
ρ	density [kg m^{-3}]
Λ_K	Kolmogorov length scale [m]
θ	local azimuthal angle in the pipe's cross section [°]
Θ	global azimuthal angle around the curvature axis [°]
τ_w	shear stress [Pa]
ϕ	generic scalar quantity

Subscripts

av	cross sectional average
b	bulk
cr	critical
RAD	radial
rms	root mean square
s, r, θ	streamwise (axial), radial and azimuthal directions
T	thermal
w	wall
θ	azimuthal

Superscripts

+	expressed in wall units

Averages

$\langle\rangle$	azimuthal- or wall-average
$\overline{()}$	time average
$'$	fluctuating component

$\langle \bar{q}''_w\rangle = \rho c_p u_\tau T_\tau$. The dimensionless temperature T^+ will be computed as $(T_w - T)/T_\tau$; note that T^+ vanishes at the wall and is always positive in the bulk flow, provided the wall heat flux q''_w is regarded as positive if entering the fluid and negative otherwise.

The friction velocity will also be used to define the Large Eddy TurnOver Time (LETOT) a/u_τ, a scale currently used in direct and large-eddy simulations of turbulence to identify the minimum simulation time required for statistical significance. Since the wall time scale is ν/u_τ^2, one LETOT measured in wall time units will numerically coincide with the friction velocity Reynolds number Re_τ.

Finally, the classical definition of the inner (tube) side Nusselt number will be used:

$$\mathrm{Nu} = \frac{\langle q''_w\rangle 2a}{\lambda(T_w - T_b)} \quad (1)$$

where $\langle q''_w\rangle$ is the average wall heat flux, λ is the fluid thermal conductivity, T_b is the bulk fluid temperature and T_w is the wall temperature. On the basis of the previous definitions, one also has $Nu = (2a)/(\lambda T_b^+)$.

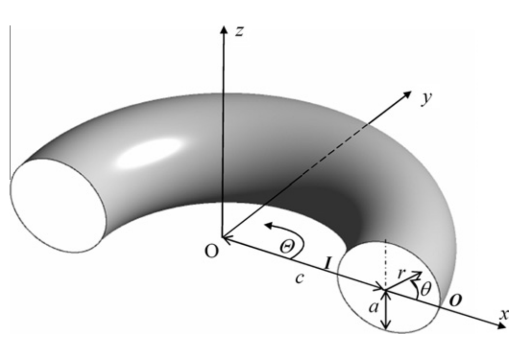

Fig. 1. Computational domain: a, pipe radius; c, coil radius; (I), (O), inner and outer sides of the curved duct; r, θ local polar coordinates in the cross-section; Θ, global azimuthal coordinate.

2. Flow and heat transfer in curved pipes and coils

2.1. Flow field and pressure drop

In curved pipes a characteristic secondary motion develops in the cross section due to the imbalance between pressure and inertial (centrifugal) forces [1]. The fluid moves towards the outer bend side near the equatorial midplane, returns towards the inner side along two near-wall boundary layers, and then forms two symmetric secondary cells (Dean vortices) having a characteristic velocity scale $u_{av}\sqrt{\delta}$.

The most popular friction correlations for curved pipes are those by Ito [4]:

$$f = \frac{64}{\mathrm{Re}} \cdot \frac{21.5 \cdot \mathrm{De}}{(1.56 + \log_{10}\mathrm{De})^{5.73}} \quad \text{(laminar flow)} \quad (2)$$

$$f = 0.304 \cdot \mathrm{Re}^{-0.25} + 0.029\sqrt{\delta} \quad \text{(turbulent flow)} \quad (3)$$

in which f is the Darcy friction coefficient (four times the Fanning friction coefficient) and Re is the bulk Reynolds number. Although It's correlations date back to the 1950s, they have been confirmed to a very high degree of approximation by recent computational [2] and experimental [5] studies.

Several authors attempted to characterize the transition to turbulence in curved pipes on the basis of friction coefficient measurements [5], flow visualization [6] or local flow/temperature measurements [7]. The main result of these studies is that curvature delays transition to turbulence with respect to straight pipes. For example, Srinivasan et al. [8] proposed the following correlation for the critical Reynolds number: