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Parametrized temperature distribution and efficiency of convective straight fins with temperature-dependent thermal conductivity by a new modified decomposition method

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ABSTRACT

In this paper, the nonlinear differential equation for temperature distribution of convective straight fins with temperature-dependent thermal conductivity is solved by using a new modified decomposition method (MDM) for boundary value problems. In the new MDM the recursion scheme of the solution components does not involve any undetermined coefficients. Using the new method, the temperature distribution and the efficiency of the fin can be expressed analytically as functions containing two fin parameters without any undetermined coefficients, which greatly facilitates parameter analysis.

1. Introduction

Fins are extensively employed to enhance the heat transfer between the primary surface and its convective, radiating or convective-radiating environment. The study of heat transfer in fins with temperature-dependent thermal conductivity is practical and essential. The governing equation of straight fins with temperature-dependent thermal conductivity is in the form of a nonlinear differential equation for which exact analytical solutions can not be obtained in general. For fin parameter analysis, approximate analytical solutions are more practical than numerical solutions.

Hung and Appl [1] presented bounds for the temperature distribution of a straight fin with temperature-dependent thermal conductivity and internal heat generation. Aziz and Huq [2] used the regular perturbation method to present a closed form solution for a straight convecting fin with temperature-dependent thermal conductivity. Muzzio [3] obtained approximate analytical solutions based on the Galerkin method, which involves selection of suitable basis functions. The Adomian decomposition method (ADM) [4–10], the homotopy analysis method [11,12], and the least squares method [13] have been used to solve the various nonlinear heat transfer models.

In this paper, we consider the nonlinear heat transfer model by a new modified decomposition method (MDM) for the nonlinear BVPs [14].

The ADM [15–24] is a well-known systematic method for practical solution of linear or nonlinear and deterministic or stochastic operator equations, and provides efficient algorithms for approximate analytical solutions and numeric simulations for real-world applications in the applied sciences and engineering. The ADM permits us to solve both nonlinear initial value problems (IVPs) and boundary value problems (BVPs) [19,25–32] without unphysical restrictive assumptions. Furthermore the ADM does not require the use of Green's functions which are not easily determined in most cases.

In [4–7] nonlinear heat transfer problems were solved by using the method of undetermined coefficients in the ADM. In [8–10] Adomian–Rach modified decomposition method, alias double decomposition method [25,26], was used for the nonlinear BVPs. The method decomposes the solution, the nonlinearity and the constants of integration, and then designs an appropriate modified recursion scheme to compute the solution components and the components of the constants of integration.

Recently Duan and Rach [14] have presented a new MDM for solving BVPs. The new modification first derives an equivalent nonlinear integral equation for the solution without any undetermined coefficients, and then by the decompositions of the solution and the nonlinearity, designs a modified recursion scheme to compute the solution components.

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The dimensionless straight fin model we consider in this work includes two fin parameters, which describe the variation of the thermal conductivity and fin structure. How the temperature distribution and the efficiency of a fin depend on the fin parameters is an important and practical issue.

In this paper we show that by the new MDM we can explicitly express the temperature distribution and the fin efficiency depending on the two fin parameters and without any undetermined coefficients.

In the next section we describe the model of the straight fin with a temperature-dependent thermal conductivity. In Section 3, we present dimensionless temperature distribution and efficiency of the straight fin with two fin parameters by the Duan–Rach MDM for BVPs. In Section 4, a comparison with the method of undetermined coefficients in the ADM is considered. Section 5 emphasizes our results.

2. Problem description

Consider a straight fin with a temperature-dependent thermal conductivity $k(T)$, arbitrary constant cross-sectional area S, perimeter P and length b. The fin is attached to a base surface of uniform temperature T_b and its tip is insulated. Under steady-state conditions, the face of the fin are exposed to a convective environment, where the temperature T_a and the heat transfer coefficient h are assumed to be uniform. Fig. 1 shows an illustration of the fin geometry, where the axial distance x is measured from the fin tip. The one-dimensional energy balance equation is given

$$S\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right] - Ph(T - T_a) = 0, \tag{2.1}$$

where T is the temperature distribution on the fin and the thermal conductivity of the fin material is assumed to be a linear function of temperature according to

$$k(T) = k_a[1 + \lambda(T - T_a)] \tag{2.2}$$

and where k_a is the thermal conductivity at the ambient temperature and λ is the parameter describing the variation of the thermal conductivity.

Employing the following dimensionless variables and parameters

$$\theta = \frac{T - T_a}{T_b - T_a}, \quad \xi = \frac{x}{b}, \quad \beta = \lambda(T_b - T_a), \quad \Psi^2 = \frac{hPb^2}{k_aS} \tag{2.3}$$

the formulation of the problem reduces to

$$\frac{d^2\theta}{d\xi^2} + \beta\theta\frac{d^2\theta}{d\xi^2} + \beta\left(\frac{d\theta}{d\xi}\right)^2 - \Psi^2\theta = 0, \tag{2.4}$$

subject to the boundary conditions

$$\left.\frac{d\theta}{d\xi}\right|_{\xi=0} = 0, \quad \theta|_{\xi=1} = 1. \tag{2.5}$$

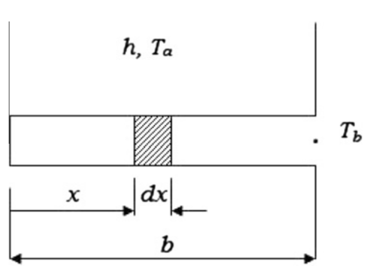

Fig. 1. Geometry of a straight fin.

We consider the ranges of the two dimensionless fin parameters: $0 < \Psi \leqslant 1.5$ and $0 \leqslant \beta \leqslant 1$.

3. Parametrized temperature distribution and efficiency by a new MDM

For convenience, we rewrite Eq. (2.4) in Adomian's operator-theoretic form

$$\mathcal{L}\theta = \mathcal{N}\theta, \tag{3.1}$$

where $\mathcal{L}(\cdot) = \frac{d^2}{d\xi^2}(\cdot)$, and

$$\mathcal{N}\theta = \frac{\Psi^2\theta - \beta(\theta')^2}{1 + \beta\theta}. \tag{3.2}$$

We define the inverse linear operator

$$\mathcal{L}^{-1}(\cdot) = \int_1^\xi \int_0^\xi (\cdot)d\xi\, d\xi, \tag{3.3}$$

then applying the inverse operator to Eq. (3.1) we obtain

$$\theta(\xi) - \theta(1) - \theta'(0)(\xi - 1) = \mathcal{L}^{-1}\mathcal{N}\theta, \tag{3.4}$$

which already involves both of the specified boundary conditions. Therefore we have

$$\theta(\xi) = 1 + \mathcal{L}^{-1}\mathcal{N}\theta. \tag{3.5}$$

We decompose the solution and the nonlinearity

$$\theta(\xi) = \sum_{n=0}^{\infty}\theta_n(\xi), \quad \mathcal{N}\theta(\xi) = \sum_{n=0}^{\infty}A_n(\xi), \tag{3.6}$$

where $A_n(\xi)$ are the Adomian polynomials, which are defined by the formula [15]

$$A_n(\xi) = \frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}\mathcal{N}\left(\sum_{k=0}^{\infty}\theta_k(\xi)\lambda^k\right)\Bigg|_{\lambda=0}. \tag{3.7}$$

The first two Adomian polynomials for the specified nonlinearity in (3.2) are

$$A_0(\xi) = \frac{\Psi^2\theta_0 - \beta(\theta_0')^2}{1 + \beta\theta_0},$$

$$A_1(\xi) = \frac{\Psi^2\theta_1 + \beta^2\theta_1(\theta_0')^2 - 2\beta\theta_0'\theta_1' - 2\beta^2\theta_0\theta_0'\theta_1'}{(1 + \beta\theta_0)^2}.$$

We note that several algorithms for symbolic programming to efficiently generate the Adomian polynomials were devised by Adomian and Rach [15], Rach [33,34], Wazwaz [35], Abdelwahid [36], Biazar and Ilie [37], Zhu et al. [38] and Azreg-Aïnou [39]. New, efficient algorithms and subroutines in MATHEMATICA for rapid computer-generation of the Adomian polynomials to high orders have been provided by Duan [40–42].

Upon substituting (3.6) into (3.5), we have

$$\sum_{n=0}^{\infty}\theta_n(\xi) = 1 + \mathcal{L}^{-1}\sum_{n=0}^{\infty}A_n(\xi).$$

Thus we have derived the recursion scheme for the solution components

$$\theta_0(\xi) = 1 - c, \tag{3.8}$$

$$\theta_1(\xi) = c + \mathcal{L}^{-1}A_0(\xi), \tag{3.9}$$

$$\theta_{n+1}(\xi) = \mathcal{L}^{-1}A_n(\xi), \; n \geqslant 1, \tag{3.10}$$

where c is a predetermined parameter, which can affect the effective region of convergence of solution approximations [14,40].