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Analysis of heatlines during natural convection within porous square enclosures: Effects of thermal aspect ratio and thermal boundary conditions

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ABSTRACT

Numerical investigation of natural convection within porous square enclosures has been performed for various thermal boundary conditions based on thermal aspect ratio on bottom and side walls. Penalty finite element analysis with bi-quadratic elements is used to solve the governing equations. The numerical solutions are studied in terms of streamlines, isotherms, heatlines, local and average Nusselt numbers for a wide range of parameters $Da(10^{-5}–10^{1})$, $Pr(0.015–1000)$ and $Ra(Ra = 10^3–10^5)$. At low Darcy number ($Da = 10^{-5}$), heatlines are perpendicular to the isotherms indicating conduction dominant heat transfer. As Da increases to 10^{-3}, convection is initiated and the thermal mixing has been observed at the central regime for all As. At low Prandtl number ($Pr = 0.015$) with high Darcy number ($Da = 10^{-2}$ and $Da = 10^{1}$), multiple circulations are observed in streamlines and heatlines and they suppressed for higher Prandtl number ($Pr = 1000$). Isotherms are highly compressed along bottom wall at higher Prandtl numbers ($Pr = 0.7$ and 1000) at $A = 0.1$ and 0.5. Temperature gradient is found to be high at the center of the bottom wall for $A = 0.1$ due to dense heatlines at that zone and that decreases as A increases from 0.1 to 0.9, irrespective of Pr, Da. Also, the temperature gradient is smaller at the top portion of side walls for $A = 0.1$ due to sparse heatlines along those zones and that is high for $A = 0.9$ due to dense heatlines. Distribution of heatlines illustrate that significant heat transport occurs from hot bottom wall to the top portion side walls at higher Darcy number ($Da = 10^{1}$). It is found that Nu_b attains maximum at $X = 0.5$ and minimum at corners for $Da = 10^{-5}$, whereas that exhibits sinusoidal variation for $Da = 10^{-3}$ and $Da = 10^{1}$ irrespective of Pr and A. It is also found that Nu_l follows wavy pattern at low Prandtl number ($Pr = 0.015$) with higher Darcy number ($Da = 10^{1}$) irrespective of A due to larger gradients of heatfunctions at several locations of left wall. The average Nusselt number show that the overall heat transfer rate is high at $A = 0.1$ compared to that of $A = 0.5$ and $A = 0.9$ irrespective of Da and Pr due to larger gradients of heatfunctions at $A = 0.1$.

1. Introduction

Natural convection within closed cavities has been extensively studied over past several years due to its wide range of applications [1]. These applications span diverse fields such as melting process [2], geothermal [3], reservoir [4], electronics cooling [5], food [6], heat exchangers [7] etc. Extensive investigations have been carried out on natural convection heat transfer by earlier researchers [8–12].

Several investigations were undertaken on natural convection within a porous square enclosures. Sankar et al. [13] studied the convective flow and heat transfer in a square porous cavity with partially active thermal walls. Badruddin et al. [14] investigated the heat transfer details in a porous square duct using finite element method. Alloui et al. [15] studied natural convection flows within a square cavity filled with binary fluid saturated porous media whereas some portion of the bottom surface is isothermally heated while the upper surface is maintained at constant cold temperature and all other surfaces are adiabatic. Varol et al. [16] analyzed the effects of diagonally inserted conductive thin plate on natural convection flow in a cavity filled with a porous medium. Mealey and Merkin [17] investigated natural convection flow within a porous square region with internal heat generation at a rate proportional to a power of the temperature difference. Pradyumna and Ghosh [18] analyzed the buoyancy driven flow under the influence of g-jitter perpendicular to the applied thermal gradient in square porous cavity.

However, most of the studies in the literature are presented in terms of streamlines and isotherms. Streamlines are generally used to explain fluid flow, isotherms are used to illustrate the temperature

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Nomenclature

A	temperature difference aspect ratio $\left(A = \frac{T_h - T_c}{T_H - T_c}\right)$ or thermal aspect ratio
Da	Darcy number
g	acceleration due to gravity, m s^{-2}
k	thermal conductivity, W m^{-1} K^{-1}
H	heatfunction
L	length of the side of the square cavity, m
Nu	local Nusselt number
$\overline{Nu}$	average Nusselt number
p	pressure, Pa
P	dimensionless pressure
Pr	Prandtl number
R	Residual of weak form
Ra	Rayleigh number
T	temperature, K
T_h	temperature at the bottom edges of the side walls, K
T_c	temperature at the top edges of the side walls, K
T_H	temperature at the center of the bottom wall, K
u	x component of velocity
U	x component of dimensionless velocity
v	y component of velocity
V	y component of dimensionless velocity
X	dimensionless distance along x coordinate
Y	dimensionless distance along y coordinate

Greek symbols

α	thermal diffusivity, m^2 s^{-1}
β	volume expansion coefficient, K^{-1}
γ	penalty parameter
θ	dimensionless temperature
ν	kinematic viscosity, m^2 s^{-1}
ρ	density, kg m^{-3}
Φ	basis functions
ψ	streamfunction

Subscripts

b	bottom wall
l	left wall
r	right wall
s	side wall

distribution in a domain, which may not be suitable to visualize the path of the heat transfer. Heatline is a useful tool to visualize the path of heat flow and also to find out the intensity of heat transfer in a domain. The heatline concept was first developed by Kimura and Bejan [19] to visualize convective heat transfer. Bejan [20] also analyzed the heatline approach for various physical situations.

A few number of articles were presented using this heatline concept for various physical phenomena. Bello-Ochende [21] formulated Poisson-type heatfunction to study heat energy distribution pattern due to natural convection within a square cavity. Dalal and Das [22] studied the natural convection heat transfer inside a two-dimensional cavity with a wavy right vertical wall where the bottom wall is heated by a spatially varying temperature and other three walls are kept at constant lower temperature. Review on Bejan's heatlines and masslines for convection visualization was also presented by Costa [23]. Recently, Varol et al. [24] analyzed the natural convection heat transfer within porous triangular enclosures with various boundary conditions. Kaluri et al. [25] investigated heat distribution and thermal mixing during natural convection within differentially heated porous square cavities based on Bejan's heatlines. Waheed [26] studied natural convection flows within porous square enclosures within rectangular enclosure whereas two horizontal walls are insulated while the left wall is hot and right walls are maintained at cold temperature.

The objective of the present investigation is to study the natural convection flows within square cavity for various boundary conditions. An important non-dimensional parameter "thermal aspect ratio (A)" is considered for thermal boundary conditions. By varying thermal aspect ratio (A) from 0 to 1, various thermal boundary conditions are imposed on the bottom and side walls, such as $A = 0$ corresponds to non-uniformly heated bottom wall and isothermal cold side walls whereas $A = 1$ corresponds to uniformly heated bottom wall and linearly heated side walls.

In the current study, we have used generalized non-Darcy model, neglecting the Forchheimer inertia term, to predict the flow and thermal characteristics in porous medium. This model based on volume averaging principles was developed by Vafai and Tien [27]. Numerical simulations were performed using Galerkin finite element method with penalty parameter to solve the nonlinear coupled partial differential equations for flow and temperature fields. The Galerkin method is further employed to solve the Poisson equation for streamfunctions and heatfunctions. The advantage of this method is that the homogeneous Neumann boundary conditions are automatically built in the formulations. The heat transfer characteristics are studied by analyzing local and average Nusselt numbers. Also, various qualitative features of local and average Nusselt numbers are adequately explained based on heatlines. Various fluids of scientific and industrial importance have been chosen for the study, namely molten metals ($Pr = 0.015$), air ($Pr = 0.7$) and olive/engine oils ($Pr = 1000$).

2. Mathematical formulation and simulation

2.1. Governing equations and boundary conditions

A schematic diagram of a two dimensional square cavity with the physical dimensions is shown in Fig. 1. The boundary conditions of velocity are considered as no-slip on solid boundaries. Confined fluid within porous bed is considered as incompressible, Newtonian and the flow is assumed to be laminar. For the treatment of the buoyancy term in the momentum equation, Bous-

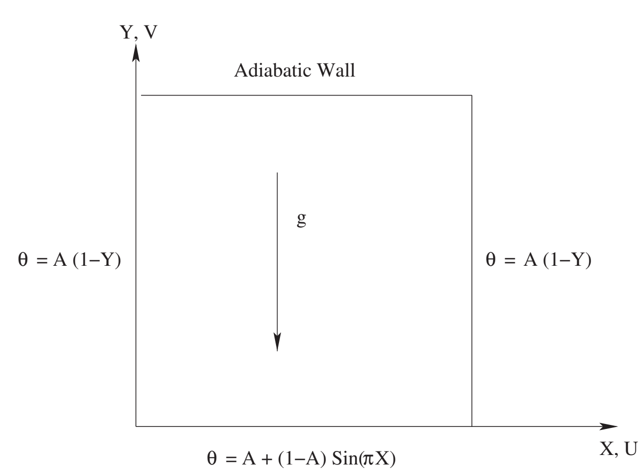

Fig. 1. Schematic diagram of the physical system.