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Forced convective heat transfer in a microtube including rarefaction, viscous dissipation and axial conduction effects

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ABSTRACT

Subsonic gas convective heat transfer in a microtube with a constant cross-sectional area and uniform wall temperature is investigated both analytically and numerically. First, the effect of rarefaction on heat transfer characteristics, at a distance from the inlet where Nu becomes constant, is analytically investigated for two cases: (*i*) including and (*ii*) neglecting the viscous dissipation effect. An exact solution for Nu in fully developed flow is presented for the case without viscous dissipation, while a closed-form solution for the asymptotic Nu is also provided for the case with viscous dissipation. Next, a numerical model is employed to investigate the simultaneous effects of rarefaction, viscous dissipation, and axial conduction for developing hydrodynamic and temperature conditions. The Nusselt number is substantially affected by viscous dissipation, rarefaction and axial conduction.

1. Introduction

Recent advances in microtechnologies require the use of small components with the gas and liquid transported through interconnected microtubes and microchannels. Furthermore, the miniaturization of electronics requires new approaches to handle higher heat fluxes. For a given volume, micro-heat exchangers, for example, provide larger and more effective heat transfer surface areas compared to conventional heat exchangers. Due in part to these advances in micro- and nanofabrication, the study of gas and liquid transport phenomena at micro- and nanoscales is receiving growing attention.

Experiments have shown that fluid flow and heat transfer in microchannels differs from that at the macroscale [1]. To this end, several theoretical and experimental investigations have been conducted to identify the physical features of microscale transport phenomena [2,3]. While it is widely accepted that gas flow through microchannels is affected by slip at the wall [1,4,5], wall slip cannot explain phenomena associated with liquid flow through microchannels. For laminar liquid flow in microtubes, both larger and smaller pressure drops (friction factors) have been reported relative to pressure drops at the macroscale. Comprehensive reviews are provided by Judy et al. [6] and Steinke et al. [7].

At the microscale, the molecular mean free path can be of the same order as the channel diameter. For gas flow, rarefaction effects at the wall, described by the Knudsen number (Kn), must be taken into account when the characteristic length is on the order of 100 μm, under normal temperature and pressure conditions. The appropriate model to describe gas flow depends on the degree of rarefaction. Different flow regimes are typically classified as follows:

1. For $Kn \leqslant 10^{-3}$ the flow is in the continuum regime.
2. For $10^{-3} < Kn \leqslant 0.1$ the flow is in the slip regime, and the Navier–Stokes equation with first or second-order slip boundary conditions should be applied.
3. For $0.1 < Kn \leqslant 10$ the flow is in the transition regime and the Boltzmann equations for dilute gases may be applied.
4. For $Kn > 10$ the flow is in the free molecular regime, and may be modeled by using the kinetic theory of gases. The governing equation is the collisionless Boltzmann equation and the direct simulation Monte Carlo approach may be employed.

The Navier–Stokes equation with first-order slip boundary conditions has been shown to be relatively accurate up to $Kn \approx 0.1$. For higher values of Kn, the Navier–Stokes equation may still be used, but with second-order slip boundary conditions and the selection of appropriate slip coefficients [8–15]. It has been shown that second-order slip boundary conditions are valid for gas flow in microchannels up to $Kn \approx 0.25$ [16], or even $Kn \approx 0.4$ [11]. However, there is a large variation in reported second-order slip coefficients [10,17]. In general, there is insufficient experimental data to validate the accuracy of many second-order models and there is no general consensus for a generic second-order slip boundary condition [15]. In this study, $(10^{-3} < Kn \leqslant 0.1)$, and the first-order slip

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Nomenclature

a	speed of sound (m s^{-1})
Br	Brinkman number ($\mathrm{Br} = \mu u_m^2 \cdot k^{-1}(T_{in} - T_w)^{-1}$)
c_p	specific heat (J kg^{-1} K^{-1})
C_f	friction factor
D	tube diameter (m)
F	tangential momentum accommodation factor
F_T	thermal accommodation factor
h	heat transfer coefficient (W m^{-2} K^{-1})
k	thermal conductivity (W m^{-1} K^{-1})
Kn	Knudsen number ($\mathrm{Kn} = \lambda \cdot D^{-1}$)
L	tube length (m)
Ma	Mach number ($\mathrm{Ma} = u \cdot a^{-1}$)
Nu	Nusselt number ($\mathrm{Nu} = hD \cdot k^{-1}$)
p	pressure (Pa)
Pe	Peclet number ($\mathrm{Pe} = u_{in}D \cdot \alpha^{-1}$)
Pr	Prandtl number ($\mathrm{Pr} = \mu c_p \cdot k^{-1}$)
q''	heat flux (W m^{-2})
r	radial direction along the radius (m)
R	tube radius (m)
Re	Reynolds number ($\mathrm{Re} = \rho u_{in} D.\mu^{-1}$)
T	temperature (K)
u	velocity in the axial direction (m s^{-1})
v	velocity in the radial direction (m s^{-1})
x	axial coordinate (m)

Greek symbols

α	thermal diffusivity (m^2 s^{-1})
β	constant
η	dimensionless radial coordinate
λ	molecular mean free path (m)
μ	dynamic viscosity (Ns m^{-2})
υ	kinematic viscosity (m^2 s^{-1})
θ	dimensionless temperature
γ	specific heat ratio ($\gamma = c_p/c_v$)
ρ	density (kg m^{-3})
σ_F	momentum slip factor
σ_{F_T}	temperature slip factor

Subscripts:

Br, ∞	corresponding to an infinite distance from the inlet with the viscous dissipation
c	centerline
in	inlet
lim	corresponding to the limit value of Ma below which the flow is incompressible
m	mean value
s	fluid property adjacent to the wall
w	solid wall
∞	value at $x \to \infty$

Superscripts:

+	non-dimensional symbol

boundary condition is utilized by making use of the kinetic theory of gases. Using this theory, the velocity jump at the wall is [18,19]:

$$u_s = -\sigma_F \lambda \left(\frac{\partial u}{\partial r}\right)_{r=R}, \quad \sigma_F = \frac{2-F}{F} \tag{1}$$

where u_s is the slip velocity at the stationary wall, λ denotes the molecular mean free path, σ_F is the momentum slip factor, and F is the tangential momentum accommodation coefficient (TMAC) which can vary from unity (complete accommodation, diffuse reflection) to zero (specular reflection). In a similar manner, the temperature jump at the wall is described by [18,19]:

$$T_s - T_w = -\sigma_{F_T} \lambda \left(\frac{\partial T}{\partial r}\right)_{r=R}, \quad \sigma_{F_T} = \frac{2-F_T}{F_T}\frac{2\gamma}{\gamma+1}\frac{1}{\mathrm{Pr}}, \tag{2}$$

where F_T is the thermal accommodation coefficient (TAC), σ_{F_T} is the temperature slip factor, T_s denotes the fluid temperature immediately adjacent to the wall, and T_w is the tube wall temperature. Experimentally-measured TMAC and TAC covering a broad range of pressures, temperatures, and materials are reported in Table 1 [16,20–27]. TMAC and TAC are nearly constant for given gas and solid surface combinations. While many investigators have assumed a value of unity for both F and F_T [28–36], experimental measurements suggest values less than unity for both. In the present study, unless otherwise noted, F and F_T are both assumed to be 0.85 [18].

Several analytical [28–30,34–38] and numerical [31–33] models have been developed to investigate heat transfer involving gas flow in both microtubes [28–31,35,36] and microchannels [32–34,37,38]. Barron et al. [28] solved the Graetz problem in the slip flow regime without considering the temperature jump at the wall. It was reported that Nu increases as Kn increases for a given Graetz number. Contradicting this conclusion, Ameel et al. [29] reported that for fully developed slip flow in a microtube with a constant wall heat flux, Nu decreases with increasing Kn. Tunc and Bayazitoglu [35] resolved the contradiction noting that Nu monotonically decreases as Kn increases, if both temperature and velocity jumps are accounted for.

In a related study, Aydin and Avci [30] analytically determined Nu in a microtube including both viscous dissipation and rarefaction effects for fully-developed hydrodynamic and thermal conditions. Because of an inappropriate definition of the Brinkman number (Br), it was concluded that for the fluid cooling case ($T_{in} > T_w$), increasing Br decreases Nu, while for fluid heating ($T_{in} < T_w$), an increase in Br increases Nu. For a constant wall temperature, they improperly assumed that the fluid temperature immediately adjacent to the wall (T_s) does not change in the flow direction. There are also other inconsistencies in their analysis [39]. For continuum flow (Kn = 0) inside a tube with a constant wall temperature and including viscous dissipation, Nu asymptotically approaches 9.60 for all values of Br [31,40,41], whereas the result of [30] does not predict this behavior.

Representative numerical studies of microscale heat transfer in the slip flow regime are [31–33]. Kavehpour et al. [33] modeled compressibility and rarefaction effects in parallel plate microchannels, for both uniform wall temperature and uniform wall heat flux conditions. They reported that both Nu and C_f are substantially smaller in the slip regime compared to continuum flows. Assuming hydrodynamically fully developed and thermally developing conditions, Cetin et al. [31] studied the effects of rarefaction, viscous dissipation and axial conduction for a constant wall temperature microtube. Hettiarachchi et al. [32] employed a three-dimensional numerical model to study developing flow and heat transfer in a microchannel with a constant wall temperature. A correlation for the fully developed friction factor was presented as a function of Kn and the channel aspect ratio.

To the authors' knowledge, there is no reported numerical solution for a subsonic gas flow in a microtube with developing hydrodynamic and temperature conditions that accounts for the