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Analyses of non-Fourier heat conduction in 1-D cylindrical and spherical geometry – An application of the lattice Boltzmann method

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ABSTRACT

This article deals with the implementation of the lattice Boltzmann method (LBM) for the analyses of non-Fourier heat conduction in 1-D cylindrical and spherical geometries. Evolution of the wave like temperature distributions in the medium is obtained, and analysed for the effects of different sets of thermal perturbations at the inner and the outer boundaries of the geometry. The LBM results are validated against those available in the literature, and those obtained by solving the same problems using the finite volume method (FVM). Results of the LBM are in excellent agreement with those reported in the literature, and with the results from the FVM. Computationally, the LBM has an advantage over the FVM.

1. Introduction

Nothing is instantaneous. The maximum speed of propagation is limited by the speed of light, i.e., $c = 3.0 \times 10^8 \mathrm{m\,s}^{-1}$. If the effects of transport of thermal radiation (light is part of the thermal radiation) are investigated at time scale of the $O\left(\frac{\text{distance}}{\text{speed of light}} \approx 10^{-9}\ \text{s}\right)$ or lower, even the transport by the fastest mode of heat transfer, i.e., radiation, becomes a transient process. Thus, a time lag Γ between the cause and its associated effects exists. If the imposition of temperature difference ΔT^* across a depth Δr^* is the cause, the effect, the propagation of energy (heat flux) by conduction at any location r^* in a medium with thermal conductivity k will be as below:

$$q^*(r^*, t^* + \Gamma) = -k\nabla T^*(r^*, t^*) \quad (1)$$

It is to be noted that the time lag Γ represents the difference in time between the appearance of a temperature gradient and induction of the corresponding heat flux. This time lag allows the system to accommodate a finite speed of propagation of thermal signals, as suggested by the theory of relativity.

In the aforementioned equation, if the thermal lag time $\Gamma\left(=\frac{\alpha}{C^2}\right) \rightarrow 0$, where α is the thermal diffusivity and C is the speed of propagation of the heat wave, Eq. (1) takes the familiar form of the governing law of heat conduction, known as Fourier's law of heat conduction.

$$q^*(r^*, t^*) = -k\nabla T^*(r^*, t^*) \quad (2)$$

Thus, the Fourier's law of heat conduction (Eq. (2)) is based on the assumption that there is no time lag between the cause and the effect. In other words, as soon as the temperature gradient is imposed, the effect of conduction heat transfer will be felt instantaneously at all locations in the medium. Therefore, the non-Fourier heat conduction equation (Eq. (1)) reduces to Fourier's law of heat conduction (Eq. (2)) if heat is assumed to propagate at an infinite speed. If the left hand side (LHS) of Eq. (1) is expanded into Taylor's series, and second and higher order terms are neglected, we get the following:

$$\Gamma\frac{\partial q^*}{\partial t^*} + q^* = -k\nabla T^* \quad (3)$$

Thus, the assumption that the effects of the perturbations is felt instantaneously at all locations does not hold true when a phenomenon is investigated at lower system time scales defined as the ratio of the characteristic length dimension to the speed of propagation of the perturbation. In most commonly encountered materials, the time lag Γ is small enough to justify the omission of higher order Taylor terms. The situation is encountered fairly often (the 'Telegrapher's equation') and has application in areas other than conduction. The hyperbolic heat conduction equation obtained by ignoring the higher order terms, has been known to be fairly accurate for materials with the large time lag too.

Almost 136 years after Fourier proposed the law of heat conduction (Eq. (2)), in 1958, Cattaneo [1] and Vernotee [2] suggested a revision to accommodate the theory of relativity. They proposed a form of heat conduction equation wherein there exists a constant thermal time lag between the cause and its effects, thus generalizing the heat conduction equation (Eq. (3)).

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Nomenclature

A	area
c_p	specific heat
C	speed of thermal wave
$\vec{e}_i$	propagation velocity in the direction i in the lattice
f_i	particle distribution function in the i direction
$f_i^{(0)}$	equilibrium particle distribution function in the i direction
k	thermal conductivity
n	index for geometry: 0 – planar, 1 – cylindrical and 2 – spherical
$\vec{r}$	position
T	non-dimensional temperature
t	non-dimensional time
Greek symbols	
α	thermal diffusivity
θ	scaled non-dimensional temperature
ζ	scaled non-dimensional time
ρ	density
τ	relaxation time
Γ	time lag
Subscripts	
ref	reference value
o	initial value
Superscripts	
$*$	dimensional quantities
(n)	*N*th order term in the Chapman–Enskog expansion

The non-Fourier heat conduction model which assumes the finite propagation speed of thermal waves has found extensive applications in the analysis and design of thermal systems [3–8]. These include heat transfer in a biological system [3], in heat transfer applications with time dependant boundary conditions [4], thermal systems irradiated with pulse-laser irradiation [5–7] and single-phase reactions [8].

Unlike Fourier conduction (Eq. (2)), the analysis becomes difficult in the non-Fourier conduction (Eq. (3)) that is based on a finite propagation speed $\left(C = \sqrt{\frac{\alpha}{\Gamma}}\right)$ of the heat front. With the former, the governing equation is parabolic and in the latter, it is hyperbolic. Thus, in the literature, heat transfer with the consideration of non-Fourier effect is frequently referred to as hyperbolic heat conduction (HHC) [9–13].

In HHC, the governing equation becomes second order in both space and time, and liable to fictional numerical oscillations at discontinuities. To deal with this mathematical complexity, researchers have used various mathematical techniques [3–27], like analytic approach with hybrid Green's function [9], space-time discontinuous Galerkin method [10], kinetic flux vector splitting scheme [11], numerical solutions of the Laplace transforms in time [12] and analytical approaches based on other intergral transforms [13–14], multiple scale technique [15], lattice Boltzmann method (LBM) [16]. Apart from the study of HHC in a planar and rectangular geometry [10–16], cylindrical and spherical geometry have also been studied [9,17–22].

In recent years, there has been a surge in the application of the LBM in the analyses of a wide range of problems in science and engineering [23]. This surge in interest is owing to the mesoscopic nature of the LBM. LBM is notably more computationally efficient than most other available computational techniques. Apart from being comparatively less expensive both in terms of CPU and memory requirement, it is also amenable to extensive parallelization. Recently, it has also found applications in heat transfer problems with and without radiation [24–25]. However, as far as its application to non-Fourier conduction (HHC) is concerned, the study is scarce. The reported work of Ho et al. [16] and Mishra et al [24–25], deal with only 1-D planar geometry.

Cylindrical and spherical geometry find many applications in science and engineering. In these geometry, researchers have studied the thermal response due to the non-Fourier conduction. More prominent of these investigations include those by Chen and Chen [17], Lu et al. [18], Cossali [19], Moosaie [20], and Lin and Chen [21]. They have used different techniques to analyse the problems. However, the LBM has not been used to study the HHC in cylindrical and spherical geometry. This work is thus aimed at the analysis of non-Fourier heat conduction in both these geometry using the LBM. In the following section, we derive a non-dimensional form of the governing equation for the general 1-D geometry. Following this, the LBM formulation for the analysis of non-Fourier conduction is presented. Using Chapman–Enskog multi-scale expansion, the equivalence of the LBM with the HHC equation is established. LBM formulation is validated next by solving the same problems using the finite volume method (FVM). Results are also compared with those available in the literature. The CPU times and the number of iterations in the LBM and the FVM are also presented. Conclusions are made at the end.

2. Formulation

When only one space dimension is considered, Eq. (3) can be written as

$$\Gamma \frac{\partial q^*}{\partial t^*} + q^* = -k \frac{\partial T^*}{\partial r^*} \tag{4}$$

In the absence of convection and volumetric radiation, the governing equation pertaining to heat transfer in 1-D geometry can be written as

$$\rho c_p \frac{\partial T^*}{\partial t^*} = -\nabla^* \cdot q^* + g^* \equiv -\frac{1}{r^{*n}} \frac{\partial (r^{*n} q^*)}{\partial r^*} + g^* \tag{5}$$

Substituting for q^* from Eq. (4) in Eq. (5), we get

$$\rho c_p \left(\Gamma \frac{\partial^2 T^*}{\partial t^{*2}} + \frac{\partial T^*}{\partial t^*} \right) = \frac{k}{r^{*n}} \frac{\partial}{\partial r^*} \left(r^{*n} \frac{\partial T^*}{\partial r^*} \right) + g^* \tag{6}$$

In the aforementioned equations, ρ is the density, c_p is the specific heat, k is the thermal conductivity, g^* is the volumetric heat generation, and for index $n = 0$, 1 and 2 the equation pertains to that for the planar, the cylindrical and the spherical geometry in which along the coordinate direction, the surface area $A \propto r^{*n}$.

In Eq. (6), if in the first term $\left(\Gamma \times \left(\frac{\partial^2 T^*}{\partial t^{*2}}\right)\right)$, either the thermal lag (relaxation time) $\Gamma \left(= \frac{\alpha}{C^2}\right) \to 0$, or $\frac{\partial^2 T^*}{\partial t^{*2}} \left(= \frac{\partial(\partial T^*/\partial t^*)}{\partial t^*}\right) \to 0$, Eq. (6) takes the form which is governed by the Fourier conduction. If $\Gamma = 0$, there is no time lag between the cause, i.e., the temperature gradient and effect, i.e., the flow of heat, which means that the effect of the cause is felt instantaneously.

With non-dimensional time t, distance r, temperature T, heat flux q and volumetric heat generation g defined as