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Rayleigh–Bénard convection in a supercritical fluid along its critical isochore in a shallow cavity

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ABSTRACT

We perform a numerical investigation of the Rayleigh–Bénard convection in supercritical nitrogen in a shallow enclosure with an aspect ratio of 4. The transient and steady-state fluid behaviors over a wide range of initial distances to the critical point along the critical isochore are obtained and analyzed in response to modest homogeneous bottom heating. On account of the fluid layer being extremely thin, density stratification is notably excluded from consideration herein, which leads to the dominating role of the Rayleigh criterion in the onset of convection. Following the Boussinesq approximation, we find the power law scaling relationships over five decades of the Rayleigh number (*Ra*) for various transient quantities including the exponential growth rate of the mean enstrophy in the cavity and the characteristic times of the development of convective motion. The correlation of the Nusselt number versus the Rayleigh number shows asymptotic features at the two ends of the *Ra* spectrum, which incidentally correspond to different convection patterns. Under the regime of high *Ra*, the heat transfer through the fluid cavity is enhanced by the turbulent bursts of thermal plumes from the boundary layers. On the other hand, under the regime of low *Ra*, it is the orderly multicellular flow that moves heat from the bottom of the layer to the top, which includes a transition from a four-cell structure to a six-cell structure with decreasing *Ra*.

1. Introduction

Rayleigh–Bénard (RB) convection (heated from below) is among the very first and most extensively studied systems concerning hydrodynamic instability [1]. Warmer fluid tends to be pushed upward by buoyancy force, and cooler fluid in turn falls down to take its place. Once the critical heat rate is reached, sustained motion arises. Linear stability analysis suggests that two limiting cases exist as regards the onset of the thermogravitational convection [2]. Under the Boussinesq approximation (BA), mechanical equilibrium is maintained by dissipative processes as compressibility is largely neglected except in the Navier–Stokes equation. Under this scenario, convection motion is characterized by the Rayleigh criterion. On the other hand, when dissipation is outweighed by compressibility, the onset of convection is determined by the Schwarzschild criterion. Near the liquid–gas critical point (CP), fluid properties exhibit unusual asymptotic behaviors. Remarkably, the thermal diffusivity ($D_T = \lambda/\rho C_P$, where λ is the thermal conductivity, ρ is the fluid density, and C_P is the specific heat at constant pressure) goes to zero and the isobaric thermal expansion coefficient α_P tends to infinity. The theoretical consideration regarding mechanical stability ought to accordingly comprise both the contributions of compressibility and dissipation. The postulated crossover between the Rayleigh and Schwarzschild criteria near the CP within the same space scale [3–5] was confirmed in a supercritical ^{3}He experiment conducted by Kogan and coworkers [6–8], where traces of the temperature difference across a flat RB cell were meticulously measured and recorded for a variety of initial temperatures along the critical isochore and imposed heat rates. (We note that in a previous experiment on the RB convection in near-critical SF_6 by Assenheimer and Steinberg [9], compressibility-induced deviation from the BA was deemed irrelevant on account of the fluid layer being extremely thin.) The ensuing theoretical [10,11] and numerical [12,13] efforts succeeded in reproducing the main findings of the experiment, which yielded good agreement for most of the results. Nevertheless, marked discrepancies between the measurements and predictions remain with regard to the transient data obtained relatively far from the CP. It was hypothesized that small parasitic thermal noises were responsible for the difference [14]. Based on such notion, a modified model that included artificial time-independent thermal fluctuations with favorable amplitude and spatial periodicity was established, use of which was shown to result in significant improved agreement with the experiment [15,16]. A semi-empirical scaling relation

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Nomenclature

a	dimensionless wavenumber, kL
A	dimensionless quantity, $P_i/\rho_i c^2$
B	dimensionless quantity, $P_i \kappa_T$
c	sound velocity (m s^{-1})
C	dimensionless quantity, $T_i \alpha_P$
C_P	specific heat at constant pressure (J kg^{-1} K^{-1})
C_V	specific heat at constant volume (J kg^{-1} K^{-1})
D_T	thermal diffusivity (m^2 s^{-1})
Fr	Froude number, Π^2/Lg
$\mathbf{g}$	gravitational acceleration (m s^{-2})
k	wavenumber (m^{-1})
L	characteristic length of the fluid (m)
Ma	Mach number, Π/c
Nu	Nusselt number, $qL/\lambda\Delta T$
P	pressure (Pa)
δP	pressure change (Pa), $P - P_i$
Pr	Prandtl number, χ/D_T
q	heat flux at the bottom plate (W/m^2)
Ra	Rayleigh number, $\alpha_P g \Delta T L^3/\chi\, D_T$
Ra_c	critical Rayleigh number
Re	Reynolds number, $L\Pi/\chi = 1$
t	time (s)
Δt	computational timestep (s)
T	temperature (K)
δT	temperature change (K), $T - T_i$
ΔT	horizontally averaged vertical temperature difference across the cavity based on the simulation results (K)
u	horizontal velocity (m s^{-1})
W	horizontal length of the cavity (m)
v	vertical velocity (m s^{-1})
$\mathbf{v}$	velocity vector (m s^{-1})
x	horizontal space variable (m)
y	vertical space variable (m)

Greek symbols

α_P	isobaric thermal expansion coefficient (K^{-1})
γ	ratio of the specific heats, C_P/C_V
Γ	aspect ratio of the cavity, W/L
δ_T	thickness of the thermal boundary layer (m)
ε	reduced temperature, $(T_i - T_c)/T_c$
ζ	bulk viscosity (Pa s)
η	shear viscosity (Pa s)
θ	scale of the temperature difference (K)
κ_T	isothermal compressibility (Pa^{-1})
λ	thermal conductivity (W m^{-1} K^{-1})
Π	characteristic fluid velocity (m s^{-1})
ρ	density (kg m^{-3})
$\delta\rho$	density change (kg m^{-3}), $\rho - \rho_i$
τ_D	thermal diffusion timescale (s), $L^2/4D_T$
τ_{PE}	piston effect timescale (s), $L^2/[D_T(\gamma - 1)]^2$
φ	exponential growth rate of the enstrophy in the cavity, as shown in Fig. 8
Ω	enstrophy (s^{-2})
χ	kinematic viscosity (m^2 s)
Φ	viscous dissipation function

Superscripts

$\sim$	dimensionless variables
(0)	zeroth-order terms in the expansion series in Eqs. (10)–(14)
(1)	first-order terms in the expansion series in Eqs. (10)–(14)

Subscripts

ad	adiabatic temperature gradient
b	Boussinesq fluids
c	critical point
D	diffusion
i	initial state
$onset$	onset of convection
p	peak
PE	piston effect

later was derived to describe the development of convection under these externally imposed perturbations [17].

Recent numerical studies unveiled more features of the convective motion. Rare sudden reversal of turbulent wind orientation (namely, large-scale circulation) more commonly known to occur at extreme Rayleigh numbers and over a particularly long period of time [18,19] was found in near-critical fluids under far less severe circumstances [20,11]. Two-dimensional (2D) and three-dimensional (3D) simulations identified general characteristics of the nonequilibrium system: hydrodynamic instability always begins with the collapse of thermal boundary layers, and the subsequent transition to fully-developed convection appears chaotic and lacking in spatial symmetry when very close to the CP [21,22]. Under certain strictly limited conditions, a reverse transition to stability of a heated fluid layer could be realized through crossing the Schwarzschild branch in the marginal stability curve [23]. Owing to the strong density stratification near the CP, phenomena of geophysical interest could find their scaled-down counterparts in laboratory-scale convection involving supercritical fluids, which makes it an ideal model to study those large-scale flows [24].

The vanishing thermal diffusivity and diverging compressibility near the CP give rise to another interesting critical phenomenon, the so-called piston effect (PE) [25–27]. The PE, driven by thermoacoustic processes, was shown to greatly accelerate thermalization of highly compressible near-critical fluids under fixed volume and nonterrestrial conditions [28–30]. The temperature relaxation in the bulk of the fluid is completed after the PE time $\tau_{PE} = L^2/[D_T(\gamma - 1)]^2$, which amounts to a significant reduction to the thermal diffusion time $\tau_D = L^2/4D_T$ [14,17]. Here L refers to the characteristic length of the fluid, and γ the ratio of the specific heats. The interaction of the PE and gravity has long attracted attention of researchers of supercritical fluids [31,32]. In a typical RB configuration, prior to the initiation of macroscopic fluid motion, the temperature distribution is heavily influenced by the PE. Thermal boundary layers are formed not only along the bottom hot plate, but near the top cold plate as well, from which thermal plumes could be generated when the convection threshold is exceeded [22,33–35]. However, other than indirectly affecting the resulting flow pattern, the PE was claimed to cause negligible impact on the fundamental dynamics of the system [2,36].

The aim of the paper is to study the evolution of hydrodynamic instability in supercritical nitrogen (as an example of near-critical behaviors) along its critical isochore in a 2D rectangular enclosure, from the onset of convection to the steady state. The various RB convection patterns over a wide range of initial distances to the CP [denoted by the reduced temperature $\varepsilon = (T_i - T_c)/T_c$, where $T_c = 126.192$ K is the critical temperature] under uniform heating