Chemical Engineering Journal 226 (2013) 227–237

Contents lists available at SciVerse ScienceDirect

Chemical Engineering Journal

journal homepage: www.elsevier.com/locate/cej

Improved population balance model for straining-dominant deep bed filtration using network calculations

Hao Yuan [a], Zhenjiang You [b,*], Alexander Shapiro [a], Pavel Bedrikovetsky [b]

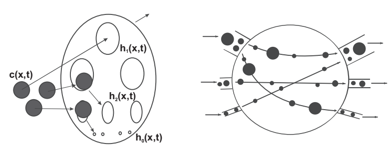
[a] Department of Chemical and Biochemical Engineering, Technical University of Denmark, DTU, Blg 229, 2800 Lyngby, Denmark
[b] Australian School of Petroleum, University of Adelaide, SA 5005, Australia

HIGHLIGHTS

- Improved population balance model for suspension transport in porous media.
- Coupled micro-scale network and large-scale population balance model.
- Correlation length of porous media is particle-size dependent.
- Combined method for correlation length determination from micro-model.
- Analysis of laboratory data.

GRAPHICAL ABSTRACT

Geometrical model for straining-dominant suspension flow in porous media.



ARTICLE INFO

Article history:
Received 11 January 2013
Received in revised form 8 April 2013
Accepted 10 April 2013
Available online 25 April 2013

Keywords:
Colloid
Suspension
Porous media
Size exclusion
Pore size distribution
Stochastic model

ABSTRACT

Colloidal-suspension flow in porous media is modelled simultaneously by the large scale population balance equations and by the microscale network model. The phenomenological parameter of the correlation length in the population balance model is determined from the network modelling. It is found out that the correlation length in the population balance model depends on the particle size. This dependency calculated by two-dimensional network has the same tendency as that obtained from the laboratory tests in engineered porous media.

1. Introduction

Straining-dominant colloidal-suspension transport in porous media occurs in numerous areas of chemical and environmental engineering. It includes industrial filtering, size exclusion chromatography, artesian wells exploitation, disposal of industrial wastes, etc. [1–5]. Often particle and pore size distributions overlap, and also the particles repel from the pore surfaces. In this case, the dominant particle capture mechanism is size exclusion (straining) where a small particle passes via a large pore, while a large particle is captured in a thin pore throat. The particle capture decreases the suspension concentration and the rock permeability [1,6,7].

Management and design of the processes involving colloid transport in porous media is based on reliable laboratory-based mathematical modelling. The classical deep bed filtration theory operates with averaged concentrations of suspended and retained particles [8–12]. The model contains the empirical filtration coefficient, which is determined from either micro-modelling or laboratory test data [10–14].

Since the capture criterion for size exclusion depends on the relationship between the particle and pore throat sizes, the adequate mathematical models should involve the probabilistic pore

* Corresponding author.
E-mail address: zyou@asp.adelaide.edu.au (Z. You).

1385-8947/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.cej.2013.04.031

Nomenclature

C	suspended particle concentration distribution by sizes, L^{-4}
c	total suspended particle concentration, L^{-3}
c_v	variance coefficient
f	fractional flow function, dimensionless
H	pore concentration distribution by sizes, L^{-4}
h	total pore concentration (density), L^{-3}
j	jamming ratio, dimensionless
l	correlation length, L
l_p	particle penetration depth, L
L	length of the core, L
N_L	size of the grid for the network
p	pressure, $ML^{-1}T^{-2}$
q	flow rate in a single pore, L^3T^{-1}
r	radius of a particle or of a pore, L
t	time, T
U	total velocity of the flux, LT^{-1}
x	linear coordinate, L
Z	coordination number, dimensionless

Greek letters

σ	total concentration of captured particles, L^{-3}
Σ	captured particle concentration distribution by sizes, L^{-4}
μ	viscosity, $ML^{-1}T^{-1}$
ϕ	porosity, dimensionless

Subscripts

a	accessible
n	inaccessible
p	pore
s	suspended (solid) particle
0	initial condition

Superscripts

0	boundary condition

and particle size distributions. The dynamics of natural pore throat and particle size distributions during flow and capture is described by population balance models [15–22]. Papers [15,16] present mass balance of suspended and captured particles with kinetic rate equations for different particle capture mechanisms; the capture system dispersivity (correlation length) is assumed to be equal to an effective pore length. Other approaches to pore and particle size distribution modelling during suspension transport in porous media include trajectory analysis [11,23,24], random walk models [22,25–29] and mean-field description of the traps [30].

Two main features of the straining-dominant suspension transport in porous media are particle flow in larger pores and their capture in smaller pores. Both features are reflected in the geometrical porous media model of parallel tubes with mixing chambers (further in the text called by its abbreviation PTMC), see [17–21]. Both processes occur in real rocks simultaneously while they are separated in PTMC model: the particle straining occurs at the chamber exits only, while the particle motion in capillaries occurs between the chambers. The cross-section in Fig. 1b corresponds to the bundle of capillaries after the first chamber in Fig. 1a. The simplified geometry of PTMC model allows for derivation of the integro-differential equations describing the suspension-colloidal transport in porous media. A major constituting parameter in these equations is the correlation mixing length that is equal to the distance between the mixing chambers. The system allows for exact upscaling from the pore scale to the core scale only in the case of mono dispersed filtration [19]. The upscaling for poly-disperse suspensions results in a separate system of partial differential equations for each particle size [22]. The upscaled equations generalise the classical large-scale system of deep bed filtration [1,8,9] introducing the physical phenomena that are specific for geometrical straining, i.e. the pore accessibility and flux reduction factor. The size exclusion factors of pore accessibility and flux reduction have been accounted for in the inlet and effluent boundary conditions, and also in the straining capture term in the equations of suspension transport in porous media [20].

The topology of the accessible sub-network strongly depends on percolation probability, i.e. on the particle size. Yet, the current population balance models assume a constant mixing (correlation) length for different size particles [15–21]. Besides the correlation length, these models contain the empirical functions of accessible fractional flow and porosity as functions of the particle radius. To the best of our knowledge, these functions have been obtained neither theoretically nor experimentally, apart from simple estimates based on the Poiseuille flow [18].

In the present work, the correlation length of the large scale size exclusion suspension flow model is obtained from a microscale network model. It is found that the correlation length in the model depends on the particle size. The properties of this dependence are analysed and explained. The particle size dependency of the correlation length as obtained from 2D network modelling and from laboratory tests with 3D flows shows the same trend.

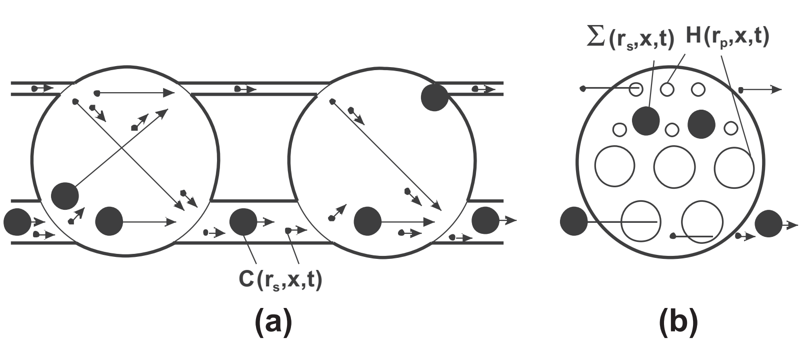


Fig. 1. Schematic for geometric model of parallel-tubes-with-mixing-chambers for porous media: (a) bundle of parallel tubes alternated with mixing chambers and (b) cross-section of a chamber.