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Scaling group transformation for MHD boundary layer free convective heat and mass transfer flow past a convectively heated nonlinear radiating stretching sheet

M. Ferdows [a,*], Md. Jashim Uddin [b,c,1], A.A. Afify [d,2]

[a] Department of Mathematics, University of Dhaka, Dhaka 1000, Bangladesh
[b] American International University-Bangladesh, Banani, Dhaka 1206, Bangladesh
[c] School of Mathematical Sciences, University Sains Malaysia, Penang 11800, Malaysia
[d] Department of Mathematics, Deanship of Educational Services, Qassim University, P.O. Box 6595, Buraidah 51452, Saudi Arabia

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ABSTRACT

Free convective heat and mass transfer flow over a moving permeable flat vertical stretching sheet in the presence of non-uniform magnetic field is numerically investigated. Effects of thermal radiation and convective surface boundary condition on steady boundary layer flow of a viscous incompressible electrically conducting fluid are considered. A scaling group of transformation is applied to the governing equations and the boundary conditions. After finding three absolute invariants a third order ordinary differential equation corresponding to the momentum equation and two second order ordinary differential equations corresponding to energy and diffusion equations are derived. Furthermore the similarity equations with the corresponding boundary conditions are solved numerically by using Runge–Kutta–Felberg fourth–fifth order numerical method. Numerical results for velocity, temperature, and concentration distributions as well as skin friction coefficient, local Nusselt, and Sherwood numbers are discussed for various values of physical parameters.

1. Introduction

Flow and heat transfer induced by a stretched surface is a relevant problem in many industrial processes such as manufacture and drawing of plastics and rubber sheets, glass-fiber and paper production, metal and polymer extrusion processes, cooling of metallic sheets in a cooling bath, crystal growing and many others. The rate of cooling plays an important role about the quality of the final product obtained from these processes, in which a moving surface emerges from a slit, and as a consequence, a boundary-layer flow is appeared in the direction of the movement of the surface. The pioneering work in this area was conducted by Sakiadis [1–3]. Sakiadis analyzed the boundary layer assumptions and the governing equations of the problem, and the boundary layer flow on a continuously stretching surface with a constant speed was investigated. The problem of flow and heat transfer over a stretching surface has been investigated and discussed by many researchers [4–7]. The study of magnetohydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. The study of MHD flow and heat transfer are deemed as of great interest due to the effect of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. Some of the engineering applications are in MHD generators, plasma studies, nuclear reactor, geothermal energy extractions, purifications of metal from non-metal enclosures, polymer technology and metallurgy. The study of the magnetohydrodynamic (MHD) flow of a boundary layer fluid over a stretching sheet has been carried out by many researchers [8–12]. Thermal radiation effects may play an important role in controlling heat transfer in industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, and power generation systems are some important applications of radiative heat transfer from a surface plate to conductive fluids. A number of studies have appeared that consider hydromagnetic radiative heat transfer flows. Interesting studies have been presented by Spreiter and Rizzi [13] in the context of solar wind radiative magnetohydrodynamics. Nath et al. [14] obtained a set of similarity solutions for radiative-MHD stellar point explosion dynamics using shooting

* Corresponding author. Permanent address: Department of Mathematics, University of Dhaka, Dhaka-1000, Bangladesh.

E-mail addresses: ferdows@du.ac.bd (M. Ferdows), afify65@yahoo.com (A.A. Afify).

1 Permanent address: American International University-Bangladesh, Banani, Dhaka 1206, Bangladesh.

2 Permanent address: Department of Mathematics, Faculty of Science, Helwan University, Ain Helwan, P.O. Box 11795, Cairo, Egypt.

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methods. Chen [15] studied the effects of anisotropic scattering on steady non-similar free convective radiative hydromagnetic boundary layer flow over a diffuse reflecting surface solving a separate equation for magnetic field distribution. Recently, Noor et al. [16] studied the effect of heat source/sink on MHD free convection thermophoretic flow over a radiate isothermal inclined plate. However, heat transfer characteristics of the stretching sheet problem have been restricted to two boundary conditions of either prescribed temperatures or heat flux at the wall in the published papers. Most recently, heat transfer problems for boundary layer flow concerning a convective boundary condition was investigated by Aziz for the Blasius flow [17]. Similar analysis was applied to the Blasius and Sakiadis flow with radiation effects [18]. Makinde [19,20] studied the heat and mass transfer over a vertical plate with convective boundary conditions. Ishak [21] studied the steady laminar boundary layer flow and heat transfer over a stationary permeable flat plate immersed in an uniform free stream with convective boundary condition. Ishak et al. [22] studied the problem of steady laminar boundary layer flow and heat transfer over a moving flat surface in a parallel stream with convective boundary condition. Subhashini et al. [23] investigated the simultaneous effects of thermal and concentration diffusions on a mixed convection boundary layer flow over a permeable surface under convective surface boundary condition. Recently, Hayat et al. [24] studied the flow and heat transfer of Eyring Powell fluid over a continuously moving surface in the presence of convective boundary conditions. Lie group analysis, also called symmetry analysis was developed by Sophius Lie to find point transformations which map a given differential equation to itself. This method unifies almost all known exact integration techniques for both ordinary and partial differential equations Oberlack [25]. In the field of viscous fluids there is many papers dealing with aspect of group theory transformation [26–33]. A special form of Lie group transformations, known as scaling group of transformation is used in this paper to find out the full set of symmetries of the problem and then to study which of them are appropriate to provide group-invariant or more specifically similarity solutions. Motivated by the above-mentioned investigations and applications, in this paper, we investigate the combined effects of thermal radiation and convective surface boundary condition on steady Magnetohydrodynamic free convective heat and mass transfer flow past a moving permeable flat vertical stretching sheet. Finally, the similarity equations are solved numerically by using Runge–Kutta–Felberg fourth–fifth order numerical method.

2. Basic equations

We consider the steady free convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching sheet. By applying two equal and opposite forces along the x-axis, in the presence of a variable magnetic field $B(\bar{x})$ normal to the plate as shown in Fig. 1. The induced magnetic field due to the motion of the electrically conducting fluid is negligible. This assumption is valid for small magnetic Reynolds number. It is also assumed that the external electric field is zero and the electric field due to polarization of charges is negligible. It is also assumed that the pressure gradient, viscous and electrical dissipation are neglected. It is assumed that the left surface of the plate is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient h_f and T_∞ the ambient fluid temperature. The species concentration at the vertical plate is also maintained uniform C_w which is also higher than the ambient fluid concentration C_∞. Fig. 1 shows the coordinates and flow model. The fluid properties are assumed to be constant except the density in the buoyancy terms of the linear momentum equation which is

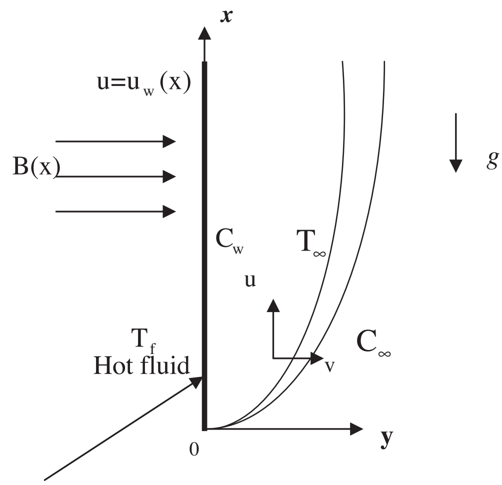

Fig. 1. Coordinate system and physical model.

approximated according to the Boussinesq's approximation. Under the above assumptions, the boundary layer form of the governing equations can be written as (see Ref. Kays and Crawford [34]):

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma_0 B_0^2}{\rho \bar{x}}\bar{u}^2 + g\beta_T(\bar{x})(T - T_\infty) + g\beta_C(\bar{x})(C - C_\infty), \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{16\sigma_1 T_\infty^3}{3\rho c_p \kappa_1}\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}$$

$$\bar{u}\frac{\partial C}{\partial \bar{x}} + \bar{v}\frac{\partial C}{\partial \bar{y}} = D\frac{\partial^2 C}{\partial \bar{y}^2}. \tag{4}$$

subject to the boundary conditions

$$\begin{aligned} &u = \bar{u}_w(\bar{x}) = c(\bar{x})^{1/3}, v = \bar{v}_w, -\kappa\frac{\partial T}{\partial \bar{y}} = h_f(\bar{x})(T_f - T(x,0)), \\ &C = C_w = C_\infty + (\Delta C)_0\left(\frac{\bar{x}}{L}\right)^m \text{ at } \bar{y} = 0, \\ &\bar{u} \to 0, T \to T_\infty C \to C_\infty \text{ as } \bar{y} \to \infty \end{aligned} \tag{5}$$

where $\bar{u}$ and $\bar{v}$ are $\bar{x}$ (along the sheet) and $\bar{y}$ (normal to the sheet) components of the velocities respectively, ρ is the density of the fluid, ν is the kinematic viscosity, $\sigma = \sigma_0\bar{u}$ is the electric conductivity, g is the acceleration due to gravity, $\beta_T(\bar{x})$ is the volumetric coefficient of thermal expansion, $\beta_C(\bar{x})$ is the volumetric coefficient of concentration expansion, c_p is the specific heat at constant pressure, α is the thermal diffusivity, m is the exponent, D is the mass diffusivity, v_w is the mass transfer velocity, σ_1 is the Sefan-Boltzman constant and κ_1 is the Rosseland mean absorption coefficient. $(\Delta C)_0$ is a constant and m is the power law index of wall temperature and concentration. A magnetic field of non-uniform strength $B(x) = \frac{B_0}{\sqrt{\bar{x}}}$ (see Helmy [35]) is applied in the y-direction normal to the plate surface. Also c is positive dimensional constant stand for characteristic stretching intensity and the dimension of c is the dimension of $(\frac{\nu}{L^{4/3}})$.

2.1. Nondimensionalization

Introducing the following dimensionless variables:

$$x = \frac{\bar{x}}{L}, y = \frac{\bar{y}}{L}, u = \frac{\bar{u}}{L}\nu, v = \frac{\bar{v}}{L}\nu, \theta = \frac{T - T_\infty}{\Delta T}, \varphi = \frac{C - C_\infty}{\Delta C}, \Delta T = T_f - T_\infty, \Delta C = C_w - C_\infty. \tag{6}$$