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Constant-yield control of continuous bioreactors

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HIGHLIGHTS

- We generalized the stability properties of chemostats under constant yield-control.
- A Lyapunov function approach was used to derive the stability region.
- Studied the effect of biomass decay and endogenous metabolism on stability.
- The stability region was greatly enlarged compared to the process under no control.
- We derived non-conservative estimates of the stability region for all cases.

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ABSTRACT

The process of anaerobic digestion is a common application in the treatment of industrial and communal wastewater treatment plants. The biochemical reactions taking place inside the fermenter can be described by a system of first order differential equations of the biomass and the limiting substrate. Biogas, a byproduct of the anaerobic digestion process is a useful energy source and thus wastewater treatment plants usually aim at maximizing its productivity yield. When the fermenter operates near optimal conditions that maximize biogas production rate though, the process is marginally stable. The intuitive idea of constant-yield control is known to improve the stability properties of the anaerobic digestion process by enlarging the stability region. In the present study, we develop a mathematical formulation of constant-yield control in the presence of biomass decay and endogenous metabolism, with main emphasis on stability analysis. In particular, we derive concrete formulas to estimate the size of the stability region of the closed loop system, and demonstrate the effectiveness of the proposed control strategy in enlarging the stability region.

1. Introduction

Continuous stirred microbial bioreactors, often called chemostats, cover a wide range of applications; specialized "pure culture" biotechnological processes for the production of specialty chemicals (proteins, antibiotics, etc.) as well as large-scale environmental technology processes of mixed cultures such as wastewater treatment. The dynamics of the chemostat is often adequately represented by a simple dynamic model involving two state variables, the microbial biomass x and the limiting organic substrate s.

A general model for chemostat dynamics is of the form (see e.g. [1,2]):

$$\begin{aligned} \frac{dx}{dt} &= -Dx + \mu(s)x - K_d x \\ \frac{ds}{dt} &= D(S_0 - s) - \frac{1}{Y_{x/s}}\mu(s)x - mx \end{aligned} \tag{1}$$

where D is the dilution rate, S_0 is the feed substrate concentration, $Y_{x/s}$ is a biomass yield factor, K_d is the biomass decay rate constant, m is the endogenous metabolism rate constant and $\mu(s)$ is the specific growth rate, a given function of s. The most widely used expressions for the specific growth rate are:
the Andrews or Haldane equation (Eq. (2a))

$$\mu(s) = \frac{\mu_{\max} s}{K_S + s + \frac{s^2}{K_I}} \tag{2a}$$

and the Monod equation (Eq. (2b))

$$\mu(s) = \frac{\mu_{\max} s}{K_S + s} \tag{2b}$$

where $\mu_{\max}$ is the maximum specific growth rate, K_S the saturation constant and K_I the substrate inhibition kinetic constant, with the

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Monod kinetics being a special case of the Andrews $\left(\frac{1}{K_I}=0\right)$ for biological reactions where substrate inhibitory phenomena can be neglected.

The problem of the chemostat stabilization has received much attention in the literature [3–9]. In the vast majority of the cases, the control input is the dilution rate D. Usually, the objective of control is to regulate the system at specific design conditions, which will net optimal performance.

One important class of applications taking place in a chemostat is related to the anaerobic digestion process, which is a key process in wastewater treatment, sludge management, production of energy from biomass, etc. During the process of anaerobic digestion, the organic compounds are mineralized to biogas, a useful energy product, consisting primarily of methane and carbon dioxide, through a series of reactions mediated by several groups of microorganisms. Under normal (balanced) operation, the rate of production of the intermediates is matched by their consumption rate; hence there is very little accumulation of these compounds. However, disturbances such as an increase in the concentration of organic compounds in the feed (organic overload), an increase in feed flow rate (hydraulic overload), presence of toxins in the feed, and temperature fluctuations, can cause an imbalance in the process [10], which results in accumulation of volatile organic acids. These acids cause a drop in the pH, inhibiting methanogenesis and eventually the reactor fails. Such a failure has major consequences in the process economics since digester recovery can be a very cumbersome and costly process. For this reason, the development of appropriate control schemes for anaerobic digesters has received significant attention in the literature [5,11–19].

For the description of the dynamics of anaerobic digestion, the mathematical model (1) can be used. This system of equations describes methanogenesis, the ultimate step in anaerobic digestion, which is rate limiting and is usually the most sensitive step. In other words, it is assumed that the bioconversion of organics into fatty acids (hydrolysis and acidification) exhibits fast equilibrium kinetics.

In anaerobic digestion, the measured output of the system is the methane production rate

$$Q = Y_m \mu(s) x \tag{3}$$

where Y_m is the yield coefficient for methane production.

In the present study, we develop a mathematical formulation of constant-yield control for system (1) and we investigate the effect of the control strategy in enlarging the stability region of the continuous stirred biochemical reactor. Although the framework is developed around a widely used application, it can easily be generalized for a wider range of systems exhibiting similar dynamic behavior. In Section 2 we examine the equilibrium and stability properties of the open-loop system (1), calculate the optimal operating conditions where the system must be regulated, and explain the nature of the control problem. Finally, in Sections 3–5, the constant-yield output feedback controller is applied to the process and the stability properties of the resulting closed-loop system are established via Lyapunov-function analysis [20].

2. Open-loop system: properties and optimal operating conditions

2.1. Hypothesis

Consider the dynamic system (1), with $\mu(s)$ given by (2a) or (2b), where the dilution rate D is the input variable of the system and $S_0, K_d, Y_{x/s}, \mu_{\max}, K_S, K_I, m$ are constant parameters. The following hypothesis will be made throughout this paper:

$$-mY_{x/s} \leqslant K_d < \mu(S_0) \tag{H}$$

Assumption (H) guarantees the existence of positive steady state(s) for the bioreactor ($x_s > 0$, $s_s > 0$ corresponding to $D_s > 0$).

Remark. In the present paper, the parameters m and K_d are understood to be nonnegative, which makes the left inequality of assumption (H) automatically satisfied. However, the results on the equilibrium properties (open-loop and closed-loop) to be presented in this paper do not require nonnegativity of the parameter m; small negative values of m are permitted as long as (H) is satisfied. In the stability results to be presented in Section 5, we will make an explicit additional assumption that m is nonnegative.

2.2. Equilibrium curve

The steady states of system (1) can be calculated from the set of equations:

$$\begin{aligned} x_s &= Y_{x/s}(S_0 - s_s)\frac{\mu(s_s) - K_d}{\mu(s_s) + mY_{x/s}} \\ D_s &= \mu(s_s) - K_d \end{aligned} \tag{4}$$

Hence the equilibrium curve is defined by

$$x_s = Y_{x/s}(S_0 - s_s)\frac{\mu(s_s) - K_d}{\mu(s_s) + mY_{x/s}} \iff x_s = (S_0 - s_s)\frac{1 - \frac{K_d}{\mu(s_s)}}{\frac{1}{Y_{x/s}} + \frac{m}{\mu(s_s)}} \tag{5}$$

Under assumption (H), the equilibrium curve (locus of points (x_s, s_s) with $x_s > 0$, $s_s > 0$, $D_s > 0$) has the shape shown in Fig. 1.

In Fig. 1, s_* represents the smallest root of the equation $\mu(s) = K_d$ and is given by:

$$s_* = K_S \cdot \frac{2\frac{K_d}{\mu_{\max}}}{\left(1 - \frac{K_d}{\mu_{\max}}\right) + \sqrt{\left(1 - \frac{K_d}{\mu_{\max}}\right)^2 - 4\left(\frac{K_S}{K_I}\right)\left(\frac{K_d}{\mu_{\max}}\right)^2}} \tag{6a}$$

In the special case of Monod kinetics $\left(\frac{1}{K_I}=0\right)$,

$$s_* = K_S \cdot \frac{\frac{K_d}{\mu_{\max}}}{1 - \frac{K_d}{\mu_{\max}}} \tag{6b}$$

is the unique root of $\mu(s) = K_d$.

Remark. For $\frac{1}{K_I} \neq 0$, the equation $\mu(s) = K_d$ has two roots, which are both real and positive; the smallest root s_* is given by (6a) and the largest root is

$$s^* = K_S \cdot \frac{2\frac{K_d}{\mu_{\max}}}{\left(1 - \frac{K_d}{\mu_{\max}}\right) - \sqrt{\left(1 - \frac{K_d}{\mu_{\max}}\right)^2 - 4\left(\frac{K_S}{K_I}\right)\left(\frac{K_d}{\mu_{\max}}\right)^2}} \tag{7}$$

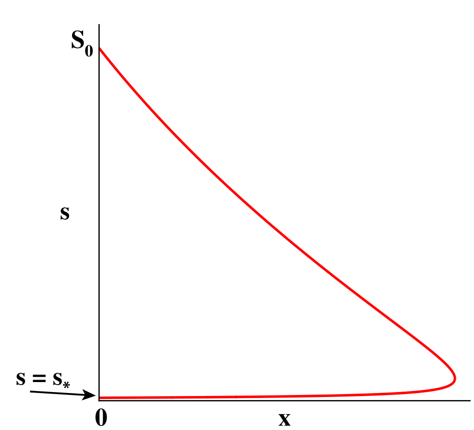

Fig. 1. Equilibrium curve of the open-loop system.