International Journal of Heat and Mass Transfer 56 (2013) 454–468

Contents lists available at SciVerse ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

DRBEM solution of free convection in porous enclosures under the effect of a magnetic field

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ARTICLE INFO

Article history:
Received 2 April 2012
Received in revised form 13 September 2012
Accepted 13 September 2012
Available online 27 October 2012

Keywords:
Free convection
Porous medium
Square enclosure
Trapezoidal enclosure

ABSTRACT

The dual reciprocity boundary element method (DRBEM) is applied for solving steady free convection in special shape enclosures filled with a fluid saturated porous medium under the effect of a magnetic field. The left and right walls are maintained at constant or different temperatures while the top and bottom walls are kept adiabatic. The effect of the external magnetic field on the flow and temperature behavior is visualized with different Rayleigh numbers *Ra*, Hartmann numbers *Ha* and inclination angle φ. The boundary only nature of DRBEM results in considerably small computational cost in obtaining numerical solution. The results are in good qualitative agreement with the available numerical results in the literature. It is found that the increase in the strength of the magnetic field causes the suppression on the motion of the fluid which points to the conductive heat transfer.

1. Introduction

In many fundamental heat tranfer analyses, convective flows in porous media have received much attention and played the central role due to the important applications in packed sphere beds, insulation for buildings, grain storage, chemical catalytic reactors, and such geophysical problems as frost heave. The underground spread of pollutants, solar power collectors, and geothermal energy systems include porous media. Books by Pop and Ingham [18], Ingham and Pop [10], Nield and Bejan [15] and Martynenko and Khramtsov [13] have representative studies on convective flows in porous media.

The heat transfer phenomenon includes the relationship of hydrodynamics, the convective heat transfer mechanism and also the electromagnetic field which may rise from the interaction of an applied magnetic force with the electrically conducting fluid. Problems arising in geophysics when a fluid saturates the earth's mantle in the presence of a geomagnetic field resemble the steady free convection in cavities filled with a porous medium saturated with electrically conducting fluid. Both the strength and the direction of applied magnetic field have a strong retarding influence on convection mode. By this way, the flow is stabilized and oscillatory instabilities are suppressed.

Natural convection in a square enclosure containing internal heat generation has important technological applications such as post-accident heat removal in nuclear reactor and underground storage of nuclear waste. The shape of the enclosure is usually square/rectangular, but in practical engineering applications such as solar collectors or heat exchangers different shaped duct constructions as trapezoidal enclosures can also be used.

There are lots of studies which are concerned with the analysis of natural convection in cavities of cross-section, square, trapezoidal, triangle or parallelogram. In some of these natural convection problems the effect of outside magnetic field is not taken into consideration. Varol et al. [23] solved natural convection in a porous trapezoidal enclosure by finite difference method using regular grid and adopting staircase-like zigzag lines for the inclined boundaries. They also studied natural convection in right-angle porous trapezoidal enclosure which is partially cooled from inclined wall using finite difference method in [24]. The same problem is solved by Baytas and Pop [3] considering trapezoidal spherical enclosure. FEM with GMRES, which is a Krylov subspace based solver, is applied to solve natural convection in trapezoidal porous enclosures by Kumar and Kumar [12] using parallel computation. The application of natural convection in partially cooled and inclined rectangular enclosures filled with porous medium is done using finite volume method based finite difference method via the SIMPLE algorithm in [16]. Similarly, laminar natural convection in a partially cooled and inclined cavities is performed using the fast false implicit transient scheme algorithm using the vorticity-stream function formulation in [7]. Baytas and Pop [2] transformed

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2012.09.019

Darcy and energy equations and solved the problem of natural convection in a porous parallelogram enclosure numerically using the ADI finite difference method. The nondimensionalized problem of the steady natural convection in a square cavity filled with a porous medium with the non-Darcy model is solved using the finite volume method by Saeid and Pop in [20]. The problem of steady natural convection flow in a triangular enclosure filled with a porous medium is solved by the finite difference method and then using successive under relaxation for the solution of algebraic equations in [22].

On the other hand, it is known that the behaviors of flow and heat transfer are considerably affected with an externally applied magnetic field. Outside magnetic field with strong intensity slows down the fluid motion with a flattening tendency in the fluid velocity. Then, the heat transfer becomes conduction dominated. So, with the variations of intensity and direction of the magnetic field it is possible to control fluid behavior and rate of heat transfer in the enclosures. Thus, some of the studies consider also the effect of a magnetic field applied outside the enclosure. Khanafer and Chamkha [11] used the control volume algorithm to solve the problem of unsteady, laminar, two-dimensional natural convection heat transfer in an inclined square enclosure filled with a fluid-saturated porous medium under the effect of a transverse magnetic field and internal heat generation. Grosan et al. [9] presented the influence of both the strength and inclination angle of the magnetic field on convective modes in a square cavity. They also studied a similar problem in the case of unsteady free convection flow in [19]. The effect of a magnetic field on steady convection in a trapezoidal enclosure filled with a fluid-saturated porous medium is figured out using finite difference method by Saleh et al. [21]. In [14], the influence of the magnetic field on the heat transfer process inside tilted enclosures for a wide range of inclination angles at moderate and high Grashof numbers is investigated by using the ADI scheme. Barletta et al. [1] illustrated that a significantly strong magnetic force tends to inhibit the flow even in the presence of a high hydrodynamic pressure gradient. An enclosure filled with a viscous and incompressible fluid for natural convection flow in the presence of a magnetic field has also been studied. Ece and Büyük [6] solved this problem by using differential quadrature method discretizing the whole enclosure by using rectangles.

In this paper, we present the DRBEM solution of natural or free convection in cavities filled with a porous medium under an externally applied magnetic field. Different cross sections as square, right-angle trapezoidal and isosceles trapezoidal are taken for enclosures, and internal heat generation effect is also considered. DRBEM has the flexibility of discretizing only the boundary of the cavity and evaluate the solution at any required interior point. This results in considerable reduction in computational work compared to domain discretization methods like FDM and FEM which is mostly used in solving these type of problems as mentioned above. DRBEM has also the advantage of calculating all the space derivatives and the unknown vorticity boundary conditions in terms of coordinate matrix directly. Numerical results are discussed with the variations of *Ra*, *Ha* and the inclination angle φ together with the average Nusselt numbers.

2. Mathematical basis

We consider the steady, two-dimensional natural convection flow in square and trapezoidal enclosures described in Figs. 1 and 2.

In the square cavity case, left and right walls are maintained at a constant temperature T_0. Furthermore, a uniform source of heat generation in the flow with a constant volumetric rate of q_0''' is considered. In the isosceles trapezoidal case, left wall is heated

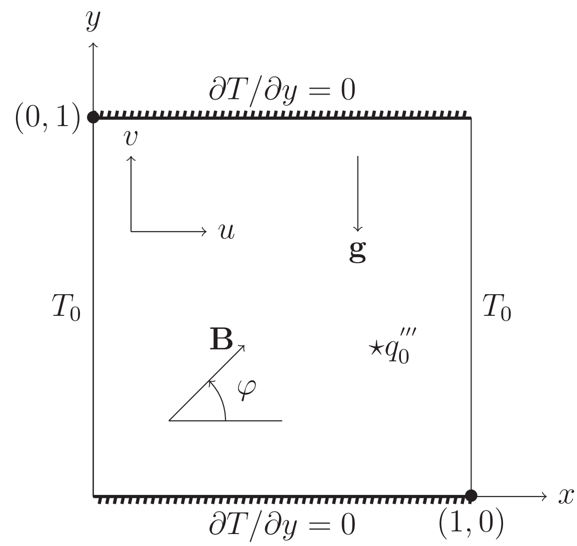

Fig. 1. Square cavity configuration.

while right wall is cooled. The partially cooled wall (in the middle) is considered while left wall is heated in the right-angle trapezoidal as is shown in Fig. 2(b). In all cases, there is an external magnetic field effect **B** at an inclination angle φ with the *x*-axis and jagged walls are adiabatic. The viscous, radiation and Joule heating effects are neglected. The combined mechanism of the driven buoyancy, internal heat generation (only for square cavity case) and the retarding effect of external magnetic field governs the resulting convective flow. Magnetohydrodynamics (MHD) is concerned with the mutual interaction of the flow of an electrically conducting and non-magnetic fluid, and the magnetic field. The applied magnetic field has the effect of slowing down the motion of the fluid. Thus, the rate of heat transfer between the walls is also accordingly changed. To be able to neglect the induced magnetic field due to the interaction of the fluid with the applied magnetic field, magnetic Reynolds number is assumed to be small. When Re_m is small, **u** has little influence on the total magnetic field **B** such that the induced magnetic field is negligible. Liquid-metal MHD generally have this property [5]. Also, $Ha = (Re\,Rh\,Re_m)^{1/2}$ may be still large due to the large values of *Re* and *Rh* where *Re* is the Reynolds number and *Rh* is the magnetic pressure of the fluid. The imposed magnetic field **B** interacting with the induced current **I** results in a Lorentz force $\mathbf{I} \times B$. This Lorentz force inhibits the motion of the fluid. The continuity equation for mass, equations of momentum under the Darcy approximation which are coupled with Maxwell's equations due to Ohm's law, and energy and electric transfer equations are coupled governing equations. With the simplified assumptions mentioned above, they are given by Grosan et al. [9]

$$\nabla \cdot \mathbf{V} = 0 \tag{2.1}$$

$$\mathbf{V} = \frac{K}{\mu}(-\nabla P + \rho\mathbf{g} + \mathbf{I} \times \mathbf{B}) \tag{2.2}$$

$$(\mathbf{V} \cdot \nabla)T = \alpha_m \nabla^2 T + HGP \tag{2.3}$$

$$\nabla \cdot \mathbf{I} = 0 \tag{2.4}$$

$$\mathbf{I} = \sigma(-\nabla\phi + \mathbf{V} \times \mathbf{B}) \tag{2.5}$$

$$\rho = \rho_0[1 - \beta(T - T_0)] \tag{2.6}$$

where $HGP = q_0'''/(\rho_0 c_p)$ is the heat generation parameter which is considered in the square cavity problem, and it is taken as zero in the trapezoidal cavity cases. In these equations, $\mathbf{V} = (u, v)$ is the fluid velocity vector, *K* is the permeability of the porous medium, μ is the dynamic viscosity, *P* is the pressure, ρ is the fluid density, **g** is the gravitational acceleration vector, **I** is the electric current, *T* is