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Non-linear stability analysis of pressure drop oscillations in a heated channel

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HIGHLIGHTS

- The effect of large perturbations on pressure drop oscillations (PDOs) is studied.
- Co-dimension two bifurcation analysis for PDOs is carried out.
- Subcritical Hopf bifurcation is shown for the first time in context of PDOs.
- Generalized Hopf points separating subcritical from supercritical Hopf are shown.
- The limit cycles are explained on the basis characteristic steady state curves.

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ABSTRACT

A study of non-linear stability analysis using co-dimension two (i.e., two free parameters being varied) bifurcations related to pressure drop oscillations (PDO) in a heated channel has been carried out in this paper. In the existing literature, in the context of PDO, mostly linear stability analysis is done. A few works on non-linear stability analysis are available; however, only co-dimension one bifurcation studies of PDO have been carried out in these works. However, in practice, since the inlet temperature of the coolant is also an independent operating parameter, both inlet temperature, and inlet mass flow rate need to be considered for stability analysis. It is also noted that the existing plethora of studies on PDO is limited to the prediction of supercritical Hopf bifurcation only. However, in the current study, two types of Hopf bifurcations have been identified namely subcritical and supercritical. A subcritical Hopf bifurcation exhibits unstable limit cycles, which is a signature of the existence of unstable solutions for slightly larger perturbations even in the linearly stable region. The existence of unstable solutions indicates that linear stability analysis is not sufficient to identify the overall stability behavior. Also, a generalized Hopf point on the stability boundary has been identified which denotes a boundary between subcritical and supercritical Hopf bifurcation. Furthermore, numerical simulations are carried out in different regions (both stable and unstable) of the parameter space to understand the non-linear phenomena of the system. Moreover, the bifurcations are explained in terms of the interaction of the external and internal characteristic curves of the system. The large and small amplitude cycles are shown in the characteristic curves of the system.

1. Introduction

Two-phase flow instabilities are observed in many systems from different areas of engineering and physics notably, from chemical evaporators, thermosyphon reboilers, nuclear reactors, and thermal power plants to microchannels electronics cooling system using fluid during removal of heat. These types of instabilities are unfavorable for the system operation as they may cause several problems, such as mechanical vibration, thermal oscillation, thermal fatigue, and premature critical heat flux. The extensive reviews of two-phase flow instabilities have been carried out by several authors (Kakac and Bon, 2008; Manavela Chiapero et al., 2012; Ruspini et al., 2014; Tadrist, 2007). These instabilities can be classified into two categories: static instabilities and dynamic instabilities (Kakac and Bon, 2008; Ruspini et al., 2014; Tadrist, 2007). The static instabilities are characterized by a system operating point changing from an original equilibrium to a new equilibrium. Flow excursion, boiling crisis, and flow pattern transition are some static instabilities. Ledinegg instability is a static

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Nomenclature

A	inner area of the heated channel (m^2)
d	diameter (m)
Eu	Euler number $P_o/(M^2/A^2\rho_l)$
K_1	loss coefficient
L	heater length (m)
L_1	main tank to surge tank length (m)
l_1	first Lyapunov coefficient
m	mass flow into heater tube (kg/s)
M_o	operating or steady state mass flow (kg/s)
M	mass flow rate into surge tank (kg/s)
$\bar{m}$	dimensionless mass flow rate
P_i	main tank pressure (Pa)
P_e	exit pressure (Pa)
P_o	steady pressure drop (Pa)
P_s	surge tank pressure (Pa)
$\bar{p}$	dimensionless pressure
r	ratio of compressible volume to inner volume V_o/AL
T	inlet temperature (°C)
t	time (s)
V_o	compressible volume (m^3)
ρ_l	density of liquid (kg/m^3)
τ	dimensionless time $t/V_o\rho_l/M$
ΔP	steady state pressure drop of the heated channel (Pa)

Abbreviations

DWO	density wave oscillations
GH	generalized Hopf
HEM	homogenous equilibrium model
PDO	pressure drop oscillations

Subscripts

e	exit
i	main tank
l	liquid
o	steady state
s	surge tank
t	transient

type instability where flow excursion occurs, and it is primarily associated with the interaction of internal pressure drop characteristics (demand) curve vs. external characteristics (pump) curve (Kakac and Bon, 2008; Ruspini et al., 2014; Tadrist, 2007). Whereas dynamic instabilities are generated by feedback effect between mass flow rate; vapor generation rate and pressure drop in a heated channel. The density wave oscillations (DWO), pressure drop oscillations (PDO), acoustic oscillations, thermal oscillations and parallel channel instabilities are most observed dynamic instabilities (Kakac and Bon, 2008; Manavela Chiapero et al., 2012; Ruspini et al., 2014; Tadrist, 2007).

The DWO are a relatively fast phenomenon in which period of oscillations is very low (of the order of residence time of fluid particle inside the heated channel). The DWO are one of the most studied phenomena in the context of instability in a two-phase flow system (Kakac and Bon, 2008; Ruspini et al., 2014). Several authors have carried out non-linear stability analysis in channels with different conditions and configurations. These studies have been carried out to identify supercritical and subcritical Hopf bifurcations in the system (Hu et al., 2015; Lee et al., 2016; Pandey and Singh, 2017; Papini et al., 2012; Paruya et al., 2012; Yan et al., 2017). Yan et al. (2017) have studied the DWO with ocean motions and also identified subcritical and supercritical Hopf bifurcation. Lee et al. (2016), Shankar et al. (2018) and Paruya et al. (2012) have also extensively studied non-linear stability analysis in the two-phase flow systems.

On the other hand, the PDO are a relatively slow phenomenon with periods of oscillations quite large compared to the residence time of fluid particle inside the heated channel (Kakac and Bon, 2008; Manavela Chiapero et al., 2012; Ruspini et al., 2014). The PDO have been observed by Stenning and Veziroglu (1965) in the two-phase flow systems in the presence of compressible volume when slope between internal pressure drop curve and mass flow rate into the heater section becomes negative. Manavela Chiapero et al. (2012) have reported an extensive review of PDO. In earlier studies, several experiments have been conducted to investigate PDO in the heated channel (Comakli et al., 2002; Dorao et al., 2017; Guo et al., 2001; Manavela Chiapero et al., 2014). Stenning et al. (1967) have identified system stability boundaries. Moreover, the non-linear analysis of limit cycle of PDO is carried out using numerical integration of the ODE system. Maulbetsch and Griffith (1966) have carried out linear stability analysis, and they have derived a fundamental analytical formula for the frequency of the oscillations. Ozawa et al. (1979) have investigated PDO phenomenon empirically, as well as analytically using linear stability analysis. Several numerical simulation based studies have also been carried out to understand non-linear dynamics of the PDO in the works of Cao et al. (2000); Kakaç and Cao (2009); Kuang et al. (2017); Manavela Chiapero et al. (2013). However, in these works stability boundaries have not been identified. It is seen that the PDO are relatively less studied compared to the DWO as do not occur in high-pressure systems. However, under certain conditions (e.g. low-pressure systems) PDO are more likely to occur, and hence, the current study is quite important for such cases (Guo et al., 2001; Schlichting et al., 2010). The microchannels are another case where PDO are the most common type of instability (Kuang et al., 2017; Zhang et al., 2010).

From the above discussion, it is quite obvious that most of the works in the context of PDO are limited to linear stability analysis (valid only for infinitesimally small perturbation). Unlike DWO, there has been very limited work on non-linear stability analysis of the PDO (Liu et al., 1995; Padki et al., 1992). Moreover, those works were limited to co-dimension one bifurcations (where only one parameter is varied) and hence, these studies can identify only one type of Hopf bifurcation. Therefore, in earlier works, only supercritical Hopf bifurcation was observed in case of PDO. In the present work, co-dimension two bifurcation studies are carried out, i.e., the inlet temperature and mass flow rate are simultaneously varied to obtain stability maps in parameter space. The abovementioned studies are done by MATCONT software to investigate different types of Hopf bifurcation. The GH points are identified to separate out subcritical and supercritical part of the stability map. Moreover, the dynamical behavior of different types of Hopf bifurcation is studied using numerical simulations and bifurcations are explained in the context of interaction of external press drop characteristics and internal pressure drop characteristics of the system.

2. Mathematical background of Hopf bifurcation

Bifurcation analysis can explain the different instabilities which are observed in the physical systems. A qualitative change, (i.e., nature of equilibrium point, periodic orbits, limit cycle, etc.) in solution behavior of a dynamical system while changing one or