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Spontaneous imbibition in randomly arranged interacting capillaries

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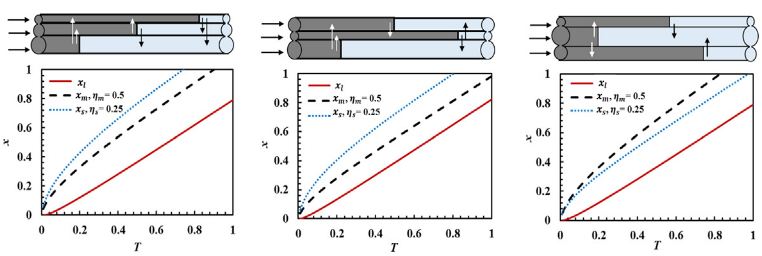
HIGHLIGHTS

- VOF simulations used to describe imbibition in an interacting two capillary system.
- Developed 1-D model for interacting capillaries using inferences from VOF simulations.
- 1-D model extended for three capillary system for random arrangement of capillaries.
- Imbibition depends on radii and arrangement of capillaries in three capillary system.

GRAPHICAL ABSTRACT



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ABSTRACT

Spontaneous imbibition in a porous medium is known to have two macroscopic fronts, the leading one in narrow pores and the lagging front in the wider pores. This behavior contradicts the behavior predicted by Washburn law due to interaction of the pores in the porous medium. In this work, we investigate the spontaneous imbibition behavior in a randomly arranged interacting capillaries system and propose a one dimensional Washburn like model which can be used to upscale the porous medium properties. We first use Volume of Fluid (VOF) simulations, which tracks the fluid-fluid interface in the multiphase flow, and show several approximations that can be used to develop a quasi one-dimensional Washburn like model for interacting capillaries. Using these approximations, we build a model for two and three capillary systems, which is able to mimic the spontaneous imbibition in a three-dimensional interacting capillary system simulated using VOF. The one-dimensional model for three capillaries shows a strong dependence of the spontaneous imbibition behavior on the arrangement of the interacting capillaries and their relative radii. We show using the one-dimensional model, that the meniscus does not always lead in the smallest radius capillary in the interacting system. We also show that the spontaneous imbibition in the whole system also deviates from the Washburn like diffusive behavior.

1. Introduction

Capillary driven flow is important for many scientific and engineering applications, for example recovery of oil from the porous matrix of a fractured reservoir (Tanino and Blunt, 2012; Zhang et al., 2014), geological CO_2 sequestration (Elenius and Gasda, 2013; Celia et al., 2015), contaminant transport in aquifers (Birdsell et al., 2015; Xiao et al., 2012; Shou et al., 2010), design of textile fabrics (Sarkar et al., 2009; Du et al., 2008) and microfluidic devices (Cate et al., 2014; Gunda et al., 2011). The foundation of the capillary driven flow was laid by Lucas (1918) and Washburn (1921), who described that the fluid imbibition in an empty capillary follows diffusion like behavior, i.e., $l^2 = Dt$. Here, l is the length imbibed by the fluid in time t and D is the effective diffusivity coefficient dependent on the capillary and the fluid properties. More precisely, $D = \frac{\gamma r}{2\mu}$ for a capillary of radius r, being imbibed by a completely wetting fluid of viscosity μ and surface tension γ. Several

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modifications of the Lucas-Washburn law have been proposed. Hultmark et al. (2011) suggested that the length traveled by the wetting fluid deviates from the diffusive dynamics, if the capillary is initially filled with a viscous fluid. Budaraju et al. (2016), Gorce et al. (2016) and Reyssat et al. (2008) modified the Washburn's law for axially varying capillaries. Waghmare and Mitra (2012) studied the imbibition in closed rectangular micro-channels, and Ouali et al. (2013) described the imbibition in non-circular open channels.

A porous medium is made up of several pores, which are non-uniform in size and shape. Therefore, imbibition dynamics in a bundle of independent capillaries of varying shapes and sizes have been used to describe the flow in porous media. Purcell (1949) used the capillary bundle model to study the capillary pressure curves in a porous medium, which were later used by Brooks and Corey to find the relative permeability for oil and gas reservoirs (Corey, 1954; Brooks and Corey, 1964). Using bundle of tubes model, Dahle et al. (2005) described the dynamic effects on capillary pressure-saturation relationship and Bartley and Ruth (1999, 2001) explained relative permeability of a porous medium. The capillaries collectively represent an ideal porous medium, which can explain the underlying dynamics of imbibition in detail. In bundle of capillaries, the meniscus in the largest radius capillary leads according to the Washburn's diffusive law. This is because, the driving force of the capillary pressure scales as $\frac{1}{r}$ and viscous resistance scales as $\frac{1}{r^2}$ for a capillary of radius r.

In the above studies, the representation of a porous medium by a bundle of capillaries ignores the hydrodynamic interaction of the pores found in naturally occurring porous media. Bico and Quéré (2003) and Dong et al. (1998) explained that the hydrodynamic interaction between the capillaries alters the imbibition dynamics significantly. In hydrodynamically interacting capillaries, the meniscus in the small radius capillary leads, contrary to the Washburn's law (Dong et al., 2005, 2006; Ashraf et al., 2017; Li et al., 2017). This behavior was seen experimentally in non-circular capillaries by Unsal et al. (2009). To develop a Washburn like one dimensional model for interacting capillaries, Dong et al. (1998) assumed that the cross-flow between the capillaries is negligible. They also assumed that in the section where capillaries are filled with the same phase, the pressure gradient in both the capillaries is same. However, their model is valid for capillaries which are arranged in the order of their radii. Ashraf et al. (2017), by analogy developed a flow model for parallel layers of packed beads assuming that pressure in the two layers is same whenever same fluid is present in both the layers, while the fluid transfer between the layers is assumed to be significant which occurs at the imbibing fluid front. However, their model neglects the viscosity of the resident fluid. In the above models, the validity of the assumptions made is still not known.

In the present work, we use Computational Fluid Dynamics (CFD) to investigate capillary driven flow in interacting capillaries. We use Volume of Fluid (VOF) method, which tracks the fluid-fluid interface, to describe the dynamics of imbibition in a hydrodynamically interacting two capillary system. Using the high fidelity simulations, we present the inferences that can be used to build a quasi one-dimensional lubrication approximation model for interacting capillaries. We develop the quasi one-dimensional model based on the inferences and then compare with the VOF simulations. The one-dimensional model sufficiently explains the invasion dynamics in a two capillary system. We further use the inferences from the VOF simulations to develop spontaneous imbibition model in an interacting three capillary system for any random arrangement of capillaries. Using the three capillary system, we show that the flow behavior is strongly dependent on the radii and the relative arrangement of the capillaries. The new model can be used to study the imbibition in porous media (Sun et al., 2016; Mullins et al., 2007).

2. VOF model for capillary driven flow

Volume of Fluid (VOF) method was introduced by Hirt and Nichols (1981) for multiphase flow. The method solves volume-averaged Navier-Stokes equation for two fluid phases and captures the fluid-fluid interface by tracking the volume fraction of a phase in a grid block. For the capillary driven flow, a capillary initially filled with a non-wetting fluid of viscosity μ_{nw} and density ρ_{nw} is considered. Then, one end of the capillary is brought in contact with a completely wetting fluid of viscosity μ_w and density ρ_w. The contact angle of the wetting phase with the capillary surface is θ in presence of the non-wetting phase. The opposite end of the capillary is assumed to be in contact with the non-wetting phase at all times, so that no capillary end effects are present at the outlet boundary. To model this phenomenon using VOF, the Navier-Stokes equation and the equation of continuity as shown in Eqs. (1) and (2) are solved for velocity and pressure of the volume averaged mixture of fluids.

$$\frac{\partial}{\partial t}(\rho\mathbf{v}) + \nabla\cdot(\rho\mathbf{v}\mathbf{v}) = -\nabla P + \nabla\cdot[\mu(\nabla\mathbf{v} + (\nabla\mathbf{v})^{\mathbf{T}})] + \mathbf{F}, \tag{1}$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{v} = 0. \tag{2}$$

Here, $\mathbf{v}$ is the velocity vector, ρ is the density, μ is the viscosity of the mixture and P is the pressure. The term $\mathbf{F}$ in Eq. (1) is the volumetric force due to capillary pressure, which is a result of the surface adhesion forces. In this study, we have considered horizontal capillaries where the effect of gravity will be negligible (Das and Mitra, 2013). All the grid-cells are occupied by either the wetting phase or the non-wetting phase except for the interface cells. The density and the viscosity for the interface cells are defined by,

$$\rho = (1-\alpha)\rho_{nw} + \alpha\rho_w, \tag{3}$$

$$\mu = (1-\alpha)\mu_{nw} + \alpha\mu_w, \tag{4}$$

where α is the volume fraction of the wetting phase. The unknown α can be solved by phase continuity equation,

$$\frac{\partial}{\partial t}(\alpha\rho_w) + \nabla\cdot(\alpha\rho_w\mathbf{v}) = 0, \tag{5}$$

The volumetric capillary force $\mathbf{F}$ in Eq. (1) is a function of the curvature of the fluid-fluid interface in the capillary. Continuum Surface Force (CSF) method by Brackbill et al. (1992) is used which defines volumetric force $\mathbf{F}$ as

$$\mathbf{F} = -\gamma\kappa\mathbf{n}, \tag{6}$$

where γ is the interfacial tension between the wetting and the non-wetting phases, $\mathbf{n} = \nabla\alpha$ is the vector normal to the interface and κ is the curvature of the interface which is defined as,

$$\kappa = \frac{1}{|\mathbf{n}|}\left[\left(\frac{\mathbf{n}}{|\mathbf{n}|}\cdot\nabla\right)|\mathbf{n}| - \nabla\cdot\mathbf{n}\right]. \tag{7}$$

At the wall boundary, the CSF method calculates the unit normal vector $\widehat{\mathbf{n}}$ by,

$$\widehat{\mathbf{n}} = \widehat{\mathbf{n_1}}\cos\theta + \widehat{\mathbf{n_2}}\sin\theta, \tag{8}$$

where θ is the contact angle, $\widehat{\mathbf{n_1}}$ and $\widehat{\mathbf{n_2}}$ are unit vectors normal and tangential to the wall, respectively.

VOF method is known to induce spurious currents in surface tension driven flows because the force due to surface tension as mentioned in Eq. (6) is obtained by calculating the curvature and