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Inverse design of thermal systems with spectrally dependent emissivities

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ABSTRACT

This work extends the inverse design methodology to systems in which the emissivities of the surfaces are dependent on the wavelength. The objective is to find the powers of the heaters to achieve uniform temperature and heat flux on the design surface. The mathematical formulation is described by an ill-conditioned system of non-linear equations, which is regularized by the TSVD method. The solution involves an iterative approach in which the radiation distribution in the spectrum bands is repeatedly guessed and corrected as the estimation of the heaters temperatures evolves. The example cases demonstrate the effectiveness of the proposed methodology.

1. Introduction

Inverse design is a technique that employs inverse analysis to find accurate design solutions, and avoid some of the trial-and-error steps that are involved in the process. For instance, in the design of heating systems for thermal processing of materials, it is usually required that the rise of temperature in the material follows a specified heating curve. However, the temperature curve can be achieved only for a specific heat flux curve on the material, as imposed by the energy balance. Mathematically, this leads to a problem in which two boundary conditions are imposed to the surface while no condition is known for the heating elements of the system. The thermal design consists of finding the placements and powers of the heating elements that can satisfy the two conditions on the processed material.

Using the conventional thermal analysis in which one condition is imposed to every element of the systems requires a trial-and-error approach. The designer chooses a configuration of heaters, and then verifies if the required heat flux on the processed material is attained. This procedure typically requires a large number of guesses to reach a satisfactory solution. In the inverse design, the two conditions are imposed on the processed material, and the required conditions on the heaters are directly determined from the mathematical model. However, the mathematical model usually becomes ill-conditioned and the system of equations may not have the same numbers of equations and unknowns. Achieving a meaningful mathematical solution can be accomplished only with the use of special techniques. For problems in which the heat transfer is governed by thermal radiation heat transfer, the inverse formulation is described by the Fredholm integral equation of the first kind, known to result in an ill-conditioned system of equations [1]. Notwithstanding these difficulties, there have been considerable advances in the inverse design in the past 10 years, including solutions that involve combined-mode heat transfer and transient processes. Some of the advances in the inverse design in radiative systems can be found in França et al. [2,3], Ertürk et al. [4,5], Daun and Howell [6], França and Howell [7], Daun et al. [8], Mossi et al. [9] and Kim and Baek [10]. Those works obtained satisfactory solutions using different solution techniques, such as regularization of the system of equations, quasi-Newton minimization, Levenberg–Marquardt method and stochastic optimization.

Despite the advances in the inverse design, there are still important areas that have not been explored. All the above solutions considered that the surfaces were gray absorbers and emitters, an assumption that allows treating thermal radiation as a linear problem on the unknown radiosities of the surface elements. However, there are cases in which the spectral dependence of the surfaces radiative properties cannot be neglected, requiring the integration of the thermal radiation in the wavelength spectrum. In the inverse design analysis, there is one additional difficulty: it is unknown how the prescribed radiative heat flux is distributed in the wavelength spectrum.

This paper considers the inverse design of a three-dimensional enclosure formed by diffuse but non-gray surfaces. The objective is to find the power inputs in the heaters so that both the prescribed temperature and heat flux are achieved in the design surface. The energy transport is governed by thermal radiation, which is solved through the discretization of the radiative balance in the wavelength bands where the emissivities of all surfaces can

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Nomenclature

$\mathbf{A}$	matrix of coefficients
$\mathbf{b}$	vector of independent terms
b_m	mth term of vector $\mathbf{b}$
$C_{\Delta\lambda_i,jd}$	band factor for design surface element jd and band $\Delta\lambda_i$
e_b	blackbody emissive power (W/m^2)
$e_{b,jh}^{(i)}$	blackbody emissive power of heater element jh obtained from the inverse analysis applied to band $\Delta\lambda_i$ (W/m^2)
$e_{\lambda b}$	spectral blackbody emissive power (W/(m^2 μm))
$f_{\Delta\lambda_i}$	fraction of the blackbody emissive power in band $\Delta\lambda_i$
$F_{j-j'}$	view factor between surface elements j and j'
H	height of the enclosure (m)
I	number of bands $\Delta\lambda_i$
J	total number of elements in the enclosure
JD	number of design surface elements
JH	number of heater elements
JW	number of wall elements
L	length of the enclosure (m)
L_d	length of the design surface (m)
p	TSVD regularization parameter
q_{design}	heat flux prescribed on the design surface (W/m^2)
q_o	radiosity (W/m^2)
$q_{o,\Delta\lambda_i}$	partial radiosity in band $\Delta\lambda_i$ (W/m^2)
q_r	radiative heat flux (W/m^2)
$q_{r,\Delta\lambda_i}$	partial radiative heat flux in band $\Delta\lambda_i$ (W/m^2)
$q_{\lambda,o}$	spectral radiosity (W/(m^2 μm))
Q_r	dimensionless radiative heat flux, $q_r/\sigma T_{\text{ref}}^4$
t	dimensionless temperature, T/T_{ref}
T	absolute temperature (K)
$\mathbf{U}$	orthogonal matrix from the singular value decomposition of matrix $\mathbf{A}$
u_{mn}	mth, nth coefficient of matrix $\mathbf{U}$
$\mathbf{V}$	orthogonal matrix from the singular value decomposition of matrix $\mathbf{A}$
$\mathbf{x}$	vector of unknowns
w_n	nth singular value in diagonal matrix $\mathbf{W}$
W	width of the enclosure (m)
$\mathbf{W}$	diagonal matrix from the singular value decomposition (SVD) of matrix $\mathbf{A}$
W_d	width of the design surface (m)

Greek symbols

ε_λ	hemispherical spectral emissivity
$\varepsilon_{\Delta\lambda_i}$	hemispherical spectral emissivity in band $\Delta\lambda_i$
$\Delta\lambda_i$	spectral band, μm
γ_{avg}	arithmetic average of the relative errors γ_{jd}
γ_{jd}	relative error of the inverse design for the design surface element jd
$\gamma_{\max}$	maximum value of the relative error γ_{jd}
σ	constant of Stefan–Boltzmann, 5.67×10^{-8} W/(m^2 K^4)

Subscripts

i	ith band of the spectrum
j	general element index
jd	design surface element index
jh	heater element index
jw	wall element index
ref	reference temperature

be assumed uniform. The resulting system of equations, assembled for each band, is ill-conditioned, which is treated by means of the truncated singular value decomposition (TSVD). The set of equations is solved by first relating the known temperatures and partial radiative heat fluxes on the design surface elements directly to the unknown partial radiosities of the heating elements. Since the energy distribution in the spectral bands is unknown a priori, the proposed methodology is based on an iterative approach. The stop criterion requires the convergence of the temperatures of the heating elements as they are computed for the different bands.

2. Problem definition and formulation

Fig. 1(a) presents a schematic view of a three-dimensional enclosure, which is formed by non-gray, diffuse surfaces. Convective heat transfer is negligible, and the interior of the enclosure does not contain a participating medium, so heat is transported solely by thermal radiation exchanges between the surfaces. For the present analysis, the design surface and the heaters are located on the bottom and top of the enclosure, although it can be readily applied to different configurations. The remaining surfaces of the enclosure are designated as walls. The length, width and height of the enclosure are represented by L, W and H. As seen in the figure, the design surface does not cover the entire base, lying at some distance from the side walls to reduce the effect of the corners. The length and width of the design surface are designated by L_d and W_d. In the examples discussed in this work, the design surface and heaters are symmetrically arranged in the enclosure, so that only one quarter of the domain needs to be simulated, as shown in Fig. 1(b).

It is considered that both the temperature and the heat flux are imposed on the design surface elements, while no thermal information (temperature or heat flux) is known for the heating elements. Since heat transfer is solely governed by thermal radiation, the imposed heat flux corresponds to the radiative heat flux, which takes into account the balance between the emission and absorption by each surface element. As will be seen, the forward formulation plays an important role in the inverse solution, so the forward formulation is first discussed. In both formulations, the enclosure is divided into finite-sized square elements for the application of the radiative energy balance. The total number of elements on the design surface, heaters and walls are, respectively, JD, JH and JW. The elements of the design surface, of the heaters and of the walls are designated by jd, jh and jw, respectively, so $1 \leqslant jd \leqslant JD$, $1 \leqslant jh \leqslant JH$ and $1 \leqslant jw \leqslant JW$. When a general relation applies to any kind of surface element, the general index j will be used.

2.1. Forward formulation

In the forward formulation, one thermal condition, the temperature or the radiative heat flux, is specified on every surface element that forms the enclosure. It is considered here that the temperature is the imposed condition; in turn, the radiative heat flux is to be determined from the radiative balance in each element. For surfaces having spectral emissivities that vary with the wavelength, the radiative balance must be set for each wavelength, and then the total quantities are obtained from the integration of the spectral quantities in the entire wavelength spectrum.

The spectral radiosity of a surface element j, $q_{\lambda,o,j}$, is given by:

$$q_{\lambda,o,j} = \varepsilon_{\lambda,j} e_{\lambda b,j} + (1-\varepsilon_{\lambda,j}) \sum_{j'=1}^{J} F_{j-j'} q_{\lambda,o,j'} \quad (1)$$

The first and the second terms of the right-hand side correspond to the spectral emissive power of the surface element and to the