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Investigating the influence of local porosity variations and anisotropy effects on the permeability of fibrous media for air filtration

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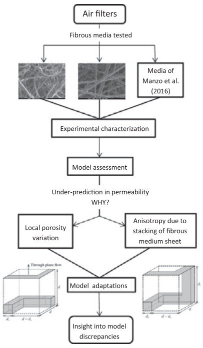
HIGHLIGHTS

- Conventional models under-predict the permeability of the fibrous filters studied.
- Local porosity variations lead to significant permeability deviations.
- An equation is proposed that investigates the effect of local porosity variations.
- A model is proposed that accounts for anisotropy due to stacking of medium sheets.

GRAPHICAL ABSTRACT



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ABSTRACT

The permeability prediction of real fibrous media remains a major challenge due to their complex porous structure. The existing models generally take two parameters into account that depict the average structural properties of fibrous media: porosity (or solid volume fraction) and fibre diameter (or radius). Due to the local variations that those parameters can exhibit the models often fail to accurately predict the permeability. In this study experimental data of three fibrous media of different porous structures employed in air filtration are used to assess numerous empirical and analytical models from the literature. It is found that all the models considered significantly under-predict the experimental permeability values. Special attention is given to the analytical Representative Unit Cell (RUC) model which is physically adaptable and can therefore be used to investigate the aforementioned discrepancies between model and experiment. Local porosity variations are investigated through a sensitivity analysis on the minimum and maximum solid volume fraction values. Incorporation of the porosity variations into the RUC model significantly reduces its under-prediction and may also serve as explanation for the discrepancies observed. An adaptation to the RUC model is proposed that takes into account the anisotropy effects as a result of the stacking of fibrous medium sheets into several layers and the increase in porosity as a result thereof. The adapted analytical model gives an indication on the factor to apply on the mean porosity in order to incorporate the anisotropy effect into the model prediction, and so reduce the observed under-predictions.

1. Introduction

The permeability of fibrous porous media depends on the medium's structural properties: fibre thickness, morphology, size

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Nomenclature

a	coefficient indicating order of mathematical dependency [–]
$A_{p\|\|}$	streamwise cross-sectional flow area in RUC [–]
B	media permeability [m^2]
d	cubic cell dimension in RUC [m]
d_e	average length of flow path in RUC [m]
d_s	rectangular solid width in RUC [m]
d_t	rectangular cell dimension in RUC [m]
D_f	fibre diameter [m]
K	modified Bessel function of the second kind [–]
r_f	fibre radius [m]
U_o	total RUC cell volume [m^3]
U_f	fluid RUC volume [m^3]
U_s	solid RUC volume [m^3]
V	superficial air velocity [m·s^{-1}]
Z	sample media thickness [m]

Greek letters

α	coefficient in Tomadakis and Robertson model [–]
γ	factor by which the cell dimension d is increased [–]
δ	fraction of fluid flowing through local porosity region [–]
ε	porosity [–]
μ	air dynamic viscosity [Pa·s]
ϕ	solid volume fraction [–]
ΔP	pressure drop [–]

Subscripts

adapt	adapted
exp	experimental
iso	isotropic
max	maximum
min	minimum
mod	modelled
sens	sensitivity
TP	through plane

distribution, diameter to length ratio and arrangement. The determination of the permeability of fibrous media for aerosol filtration (in specific) is an important aspect as this parameter influences the filter performances (i.e. pressure drop and collection efficiency). The earlier attempts in predicting the permeability of fibrous porous media were based on the assumption of isotropy. The permeability prediction assuming isotropy, i.e. B_{iso}, of Jackson and James (1986) and Davies (1952), for example, are based on empirical models and respectively given by:

$$B_{iso} = \frac{D_f^2}{4}\frac{3}{20\phi}(-\ln\phi - 0.931), \tag{1}$$

and

$$B_{iso} = \frac{D_f^2}{4\left[16\phi^{3/2}(1+56\phi^3)\right]}, \tag{2}$$

where D_f is the fibre diameter and ϕ denotes the solid volume fraction. The implicit analytical model of Spielman and Goren (1968) also based on isotropic media, is given by:

$$\frac{1}{4\phi} = \frac{1}{3} + \frac{5}{6}\frac{\sqrt{B_{iso}}}{r_f}\frac{K_1\left(\frac{r_f}{\sqrt{B_{iso}}}\right)}{K_0\left(\frac{r_f}{\sqrt{B_{iso}}}\right)}, \tag{3}$$

where K_0 and K_1 are the modified Bessel functions of the second kind and $r_f = D_f/2$ denotes the fibre radius. More recently Tomadakis and Robertson (2005) proposed the following permeability prediction for isotropic media:

$$B_{iso} = \frac{(1-\phi)D_f^2}{32\ln^2(1-\phi)}\frac{(1-\phi-\varepsilon_p)^{\alpha+2}}{(1-\varepsilon_p)^{\alpha}\left[(\alpha+1)(1-\phi)-\varepsilon_p\right]^2}, \tag{4}$$

with $\varepsilon_p = 0.037$ a percolation threshold porosity and $\alpha = 0.661$.

Due to their manufacturing processes, fibrous media can, however, exhibit anisotropic fibrous structures (e.g. Theron et al., 2017). Moreover, due to their implementation in ventilation facilities, the airflow mostly penetrates the filter through pathlines normal to the medium. Therefore, for this kind of medium the permeability is generally determined through pressure drop measurements in flat configurations normal to the flow, known as the through plane permeability. Analytical permeability predictions for through plane (and in-plane) flow were proposed for anisotropic media by e.g. Spielman and Goren (1968) Tomadakis and Robertson (2005), Tamayol and Bahrami (2010) and Van Doormal and Pharoah (2009). The following implicit analytical equation for the through plane permeability prediction (B_{TP}) was provided by Spielman and Goren (1968):

$$\frac{1}{4\phi} = \frac{1}{2} + \frac{\sqrt{B_{TP}}}{r_f}\frac{K_1\left(\frac{r_f}{\sqrt{B_{TP}}}\right)}{K_0\left(\frac{r_f}{\sqrt{B_{TP}}}\right)}. \tag{5}$$

The permeability prediction for through plane flow provided by Tomadakis and Robertson (2005) is given by the following Tomadakis and Robertson through plane model:

$$B_{TP} = \frac{(1-\phi)D_f^2}{32\ln^2(1-\phi)}\frac{(1-\phi-\varepsilon_p)^{\alpha+2}}{(1-\varepsilon_p)^{\alpha}\left[(\alpha+1)(1-\phi)-\varepsilon_p\right]^2}. \tag{6}$$

with $\varepsilon_p = 0.11$ and $\alpha = 0.785$.

Tamayol and Bahrami (2011) have classified the through plane and in-plane permeability models as representative of that of 2D fibrous media. In such media all the fibres lie in planes that are parallel to each other, but may take on random orientations, albeit still classified as anisotropic because of the 2D configuration. Several models have also been proposed in the literature for flow through 1D fibrous media (Woudberg, 2017) in which the fibres also lie in parallel planes but with their axes directed parallel with respect to each other. In such a case the flow is regarded as parallel or perpendicular (or transverse) to the fibre axes and anisotropic. The first analytical models for flow parallel and perpendicular to 1D fibrous media were proposed by e.g. Happel (1959), Drummond and Tahir (1984) and Gebart (1992). Model predictions by Tomadakis and Robertson (2005) and Tamayol and Bahrami (2010) followed in more recent years.

The isotropic permeability models mentioned above fall in the category of 3D fibrous media. An alternative analytical approach in producing 3D isotropic permeability models, apart from the models of Spielman and Goren (1968) and Tomadakis and Robertson (2005), discussed above, is by allocating a weight of 1/3 to the permeability prediction for flow parallel to 1D fibrous