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Absolute instability induced by Marangoni effect in thin liquid film flows on vertical cylindrical surfaces

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HIGHLIGHTS

- A thin film model which is applicable to the interior/exterior flow case is derived.
- An analytic bound which separates the absolute instability regime and the convective instability regime is obtained.
- The flow instability is always absolute when a composite Marangoni number exceeds a critical value (≈0.71).

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ABSTRACT

This paper investigates a thin liquid film flowing down the interior or exterior surface of a vertical uniformly heated cylinder under the influence of gravity. A thin liquid film model, which is applicable to both cases, is derived to examine the Marangoni effect on the spatial-temporal dynamics. Linear stability analysis predicts that an absolutely unstable mode could be initiated by the Marangoni effect even if the film thickness is very thin compared to the cylinder's radius. The linear stability analysis shows that the instability is always absolute for arbitrary capillary number if a composite Marangoni number $M = \frac{3MaBi}{2(1+Bi)^2}$ exceeds a critical value $M = \sqrt{\frac{-17+7\sqrt{7}}{3}} \approx 0.71$ (*Ma* is the Marangoni number, and *Bi* is the Biot number). Direct numerical simulations of the linearized and the full thin film model demonstrated the linear analysis. Results of the direct numerical simulations also show that the film has a strong tendency to break up into more droplets or rupture in the absolute instability regime. Nonlinear study also shows that the coalescence of droplets/ring waves and bound state are weakly dependent on the absolute or convective instability.

1. Introduction

Cylindrical surfaces coated with liquid films are widely encountered in nature and industrial processes, for example, viscous beads on spider silk, coating molten insulating layers on metal wires and lubricating films on the interior surfaces of pipes. Most industrial processes are operated in non-isothermal circumstances, for example, pulling out a cylinder from a hot liquid bath (Johnson and Conlisk, 1987) or cooling of hot glass fibers (Sweetland and Lienhard, 2000).

Many previous theoretical and experimental studies of the coating flows on the exterior surfaces of vertical cylinders, however, have focused on isothermal systems. Such an isothermal thin liquid film flowing along a vertical cylinder driven by gravity exhibits many interesting dynamical phenomena, e.g. formation of viscous beads, breakup of the liquid film and steady organized traveling wave states (Quéré, 1999, 1990). The well-known Plateau-Rayleigh mechanism leads to the formation of small viscous beads, while gravity is the cause of organized sliding beads. For studies of the dynamics of thin film flows on vertical cylinders, long-wave models were adopted, such as the thin liquid film models (Frenkel, 1992; Kalliadasis and Chang, 1994; Chang and Demekhin, 1999), the thick liquid film models (Kliakhandler et al., 2001; Ding and Liu, 2011), the asymptotic models (Craster and Matar, 2006; Ding et al., 2014), the integral boundary layer model (Sisoev et al., 2006) and the weighted residual model (Ruyer-Quil et al., 2008). An important theoretical finding in these previous studies is that large-sized droplets move at higher speeds which catch up with small droplets before consuming them

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(Craster and Matar, 2006; Ding et al., 2014; Sisoev et al., 2006; Ruyer-Quil et al., 2008). Experimental study of thin liquid film flows on a vertical cylinder has shown that two different flow patterns could exist in the system: an absolutely unstable flow and a convectively unstable flow (Duprat et al., 2007). It was reported that the instability is convective at large or small film thicknesses compared with the fiber's radius, and the absolutely unstable mode was observed in an intermediate range of film thicknesses (Duprat et al., 2007). An artificial noise was introduced at the inlet of the experimental setup (Duprat et al., 2007). When the flow is absolutely unstable, beads are observed in the whole experimental zone. When the flow is convectively unstable, a flat and stable interface is observed in the upstream, while the downstream is unstable and beads are only observed therein. The absolutely unstable mode is self-sustaining and exhibits a globally unstable behavior. Convectively unstable mode behaves as an amplifier of incoming perturbations which grows in the downstream region and eventually flows out of the domain. Recently, the heat transfer effect on the formation of viscous beads and wave dynamics has been taken into consideration by an asymptotic model (Liu and Liu, 2014) and a thin liquid film model (Ding and Wong, 2017). The Marangoni effect was found to enhance the Plateau-Rayleigh mechanism and could cause the rupture of the film (Liu and Liu, 2014; Liu et al., 2017). It was also found that the droplet size and the moving speed can be promoted by the thermocapillary effect (Liu and Liu, 2014). Large droplets are unstable and will break up into small droplets. The small droplets may also be unstable subjected to the non-axisymmetric disturbances and evolve into an asymmetric state (Ding and Wong, 2017). Very recently, the effect of wall slippage was investigated which reported that the slippery boundary condition enhances not only the Rayleigh-Plateau instability but also the Marangoni instability (Chao et al., 2018).

All the aforementioned studies were concentrated on the interfacial dynamics of liquid film flows on the exterior surface of a cylinder (Frenkel, 1992; Kalliadasis and Chang, 1994; Chang and Demekhin, 1999; Kliakhandler et al., 2001; Ding and Liu, 2011; Craster and Matar, 2006; Ding et al., 2014; Sisoev et al., 2006; Ruyer-Quil et al., 2008; Duprat et al., 2007; Liu and Liu, 2014; Liu et al., 2017; Ding and Wong, 2017; Chao et al., 2018). The interior coating flow is typically seen in the core-annular flows, where the gas phase occupies the core and the pipe's wall is coated by a viscous liquid layer. A distinct difference between the interior flow and the exterior flow is that the gas phase may be "choked" in the gas-liquid core-annular flow system. Experimental and theoretical studies of an air-driven core-annular flow also showed that the ring wave in the interior flow plays a key role in mass transport (Camassa et al., 2012). Furthermore, linear and nonlinear analyses by an asymptotic and a thin-film model predict the existence of an absolutely unstable mode, which causes the choke of the gas phase (Camassa et al., 2014; Camassa and Ogrosky, 2015). Very recently, the asymptotic model was applied to examine the stability of a liquid film coating the interior surface of a vertical cylinder with a porous wall (Liu and Ding, 2017). It showed that the wall-slippage promotes the absolute instability (Liu and Ding, 2017). To the best of our knowledge, there is very limited studies on the stability of gravity-driven heated films coating on the interior or exterior surface of a vertical tube. More importantly, it is unknown how the Marangoni effect affects the absolute and convective instabilities in this system. A very recent study showed that the Marangoni effect suppresses the instability of the thin film flow coating the interior surface of a uniformly heated rotating horizontal cylinder (Kumawat and Tiwari, 2017). This is a wrong conclusion because of the incorrect energy balance condition at the interface which was corrected recently (Ding and Liu, 2017; Kumawat and Tiwari, 2017). In this paper, we aim to give a unified description of 'how the Marangoni effect influences the instability in thin liquid film flowing down the interior or exterior surface of a vertical cylinder'.

The rest of the paper is organized as follows. Section 2 formulates the mathematical model. A thin film model, which is applicable to both the interior and exterior flows on the cylindrical surface, is derived in Section 3. The spatial-temporal instability is discussed in Section 4 and the direct numerical simulations of the linearized and full thin film model are carried out to illustrate the thermocapillary effect on the spatial-temporal dynamics in Section 5. A conclusion is made in Section 6.

2. Mathematical formulation

We consider a thin Newtonian liquid film flowing down the interior surface of a vertical cylinder which is heated uniformly as shown in Fig. 1. In this paper, the dynamics of gas is neglected. Here, the axisymmetric problem is considered and the cylindrical coordinates (r, z) are chosen. The motion of liquids is governed by the continuity equation and the momentum equation as below,

$$\nabla \cdot \boldsymbol{u} = 0, \tag{1}$$

$$\rho \frac{D\boldsymbol{u}}{Dt} = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \rho \boldsymbol{g}, \tag{2}$$

where $\boldsymbol{u} = u\boldsymbol{e}_r + w\boldsymbol{e}_z$ is the velocity. $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}$ is the material derivative operator. ρ is the density of the liquid and $\mu = \rho\nu$ is the dynamical viscosity where ν is the kinematic viscosity. $\boldsymbol{g}$ denotes the gravity acceleration.

The temperature field T within the liquid film is governed by the Fourier equation,

$$\frac{DT}{Dt} = k_{th} \nabla^2 T, \tag{3}$$

where k_{th} is the thermal diffusivity of the liquid.

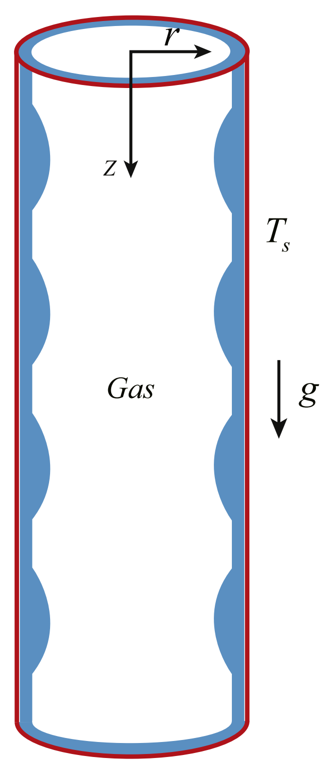

Fig. 1. Geometry of the system.