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Pressure corrections for the potential flow analysis of Kelvin–Helmholtz instability with heat and mass transfer

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ABSTRACT

Pressure corrections for the viscous potential flow analysis of Kelvin–Helmholtz instability at the interface of two viscous fluids have been carried out when there is heat and mass transfer across the interface. Both fluids are taken as incompressible and viscous with different kinematic viscosities. In viscous potential flow theory, viscosity enters through normal stress balance and effect of shearing stresses is completely neglected. We include the viscous pressure in the normal stress balance along with irrotational pressure and it is assumed that this viscous pressure will resolve the discontinuity of the tangential stresses at the interface for two fluids. It has been observed that heat and mass transfer has destabilizing effect on the stability of the system. A comparison between viscous potential flow (VPF) solution and viscous contribution to the pressure for potential flow (VCVPF) solution has been made and it is found that the effect of irrotational shearing stresses stabilizes the system.

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1. Introduction

When two superposed fluid layers of different physical parameters move parallel to each other with a relative horizontal velocity, the instability of the plane interface between the two fluids is called Kelvin–Helmholtz instability [1,2]. The Kelvin–Helmholtz instability occurs in various situations such as wind blowing over the ocean, meteor entering the earth atmosphere and in oil exploration industry etc.

When two fluids are divided by an interface, the interfacial instability is usually discussed without considering heat and mass transfer across the interface. On the other hand, the transfer of mass and heat across the interface is very important in many situations such as boiling heat transfer in chemical engineering and in geophysical problems. Hsieh [3,4] formulated the problem of Rayleigh–Taylor instability and Kelvin–Helmholtz instability with heat and mass transfer across the liquid vapour interface. Hsieh [4] found that when the vapor layer is hotter than the liquid layer, the effect of heat and mass transfer tends to inhibit the growth of instability. Nayak and Chakrborty [5] established the formulation of Kelvin–Helmholtz instability of the cylindrical interface between the liquid and vapor phases with heat and mass transfer. Lee [6] has studied the Kelvin–Helmholtz instability of inviscid fluids taking heat and mass transfer into the account and observed that the heat and mass transfer has no effect on the linear inviscid analysis while it plays an important role in the nonlinear analysis.

Viscous potential flow theory has played an important role in studying various stability problems. Tangential stresses are not considered in viscous potential theory and viscosity enters through normal stress balance [7]. The no slip condition at the boundary is not enforced in viscous potential theory. The viscous potential flow analysis of Kelvin–Helmholtz instability has been studied by Funada and Joseph [8]. They have observed that the stability criterion for viscous potential flow is given by the critical value of relative velocity. Funada and Joseph [9] studied the viscous potential flow analysis of capillary instability and observed that viscous potential flow is better approximation of the exact solution than the inviscid model. Funada and Joseph [10] extended their work of capillary instability for viscoelastic fluids of Maxwell type and observed that the growth rates are larger for viscoelastic fluids than for the equivalent Newtonian fluids.

In the viscous potential flow theory tangential stresses are not considered and viscosity enters through normal stress balance. Wang, Joseph and Funada [11] presented the idea that there exist viscous pressure along with irrotational pressure in the normal stress balance and it is assumed that this viscous pressure will resolve the discontinuity of tangential stresses for two fluids at the interface. Wang, Joseph and Funada [12] carried out the viscous contributions to the irrotational pressure for potential flow analysis of capillary instability taking a viscous fluid and another fluid of negligible viscosity to resolve the discontinuity of the tangential velocity and shear stress at the interface. The effect of irrotational

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shearing stresses on the viscous potential flow analysis of Kelvin–Helmholtz instability of two viscous fluids has been studied by Awasthi et al. [13]. They have observed that the irrotational shearing stresses stabilize the system.

The viscous potential flow analysis of Kelvin–Helmholtz instability with heat and mass transfer has been carried out by Asthana and Agrawal [14]. They observed that the heat and mass transfer has a stabilizing effect when the lower fluid viscosity is high and destabilizing effect when fluid viscosity is low. Kim et al. [15] investigated the capillary instability problem of vapour liquid system in an annular configuration with heat and mass transfer using viscous potential flow for axisymmetric disturbances. They observed that for irrotational motion of two viscous fluids, heat and mass transfer phenomenon completely stabilizes the interface against capillary effects.

The objective of the present work is to include the effects of shearing stresses in the viscous potential flow analysis of Kelvin–Helmholtz instability when there is heat and mass transfer across the interface. Both fluids are incompressible and viscous with different kinematic viscosities and having relative horizontal velocities. We have assumed that there exist viscous pressure in the normal stress balance along with irrotational pressure and this viscous pressure will resolve the discontinuity of tangential stresses, which are not in continuation of the viscous flow theory. A dispersion relation has been obtained. The dispersion relation of Asthana and Agrawal [14] has been reduced by our relation. Various graphs have been drawn showing the effects of various physical parameters such as vapor fraction, heat transfer coefficient etc. on the stability of the system.

2. Problem formulation

Consider a system of two incompressible and viscous fluid layers of finite thickness whose undisturbed interface is at $y = 0$ as demonstrated in Fig. 1. In the equilibrium state, lower fluid of density $\rho^{(1)}$ and viscosity $\mu^{(1)}$ occupies the region $-h_1 < y < 0$ and upper fluid of density $\rho^{(2)}$ and viscosity $\mu^{(2)}$ occupies the region $0 < y < h_2$. The lower and upper fluids have uniform flow $(U_1, 0)$ and $(U_2, 0)$ respectively. The bounding surfaces $y = -h_1$ and $y = h_2$ are considered to be rigid. The temperatures at $y = -h_1$, $y = 0$ and $y = h_2$ are T_1, T_0 and T_2 respectively. In the basic state, thermodynamics equilibrium is assumed and the interface temperature T_0 is set equal to the saturation temperature. In the disturbed state, the interface is given by:

$$F(x, y, t) = y - \eta(x, t) = 0 \tag{1}$$

where η is the perturbation from its equilibrium value. The unit outward normal to the first order term is given by

$$\boldsymbol{n} = \left(-\frac{\partial \eta}{\partial x}\boldsymbol{e}_x + \boldsymbol{e}_y\right) \tag{2}$$

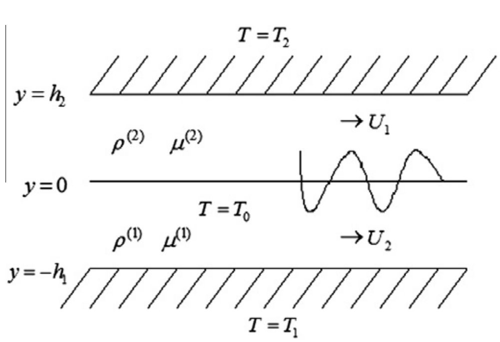

Fig. 1. The equilibrium configuration of the fluid system.

where $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ are unit vectors along x and y directions, respectively. The velocity is expressed as the gradient of a potential function and the potential functions satisfy Laplace equation as a consequence of the incompressibility constraint. That is,

$$\nabla^2 \phi^{(j)} = 0 \quad (j = 1, 2) \tag{3}$$

At the walls normal velocity vanishes, hence

$$\frac{\partial \phi^{(j)}}{\partial y} = 0 \quad \text{at } y = (-1)^j h_j \quad \text{for } (j = 1, 2) \tag{4}$$

The interfacial condition, which expresses the conservation of mass, can be written as

$$[\![\rho\left(\frac{\partial F}{\partial t} + \nabla\phi \cdot \nabla F\right)]\!] = 0 \tag{5}$$

where $[\![x]\!] = x^{(2)} - x^{(1)}$ represents the difference in a quantity across the interface. Using Eqs. (1) and (5) we get

$$[\![\rho\left(\frac{\partial \phi}{\partial y} - \frac{\partial \eta}{\partial t} - \frac{\partial \eta}{\partial x}\frac{\partial \phi}{\partial x}\right)]\!] = 0 \quad \text{at } y = \eta \tag{6}$$

The interfacial condition for energy transfer can be expressed as

$$L\rho^{(1)}\left(\frac{\partial F}{\partial t} + \nabla\phi^{(1)} \cdot \nabla F\right) = S(\eta) \quad \text{at } y = \eta \tag{7}$$

where L is the latent heat released during phase transformation. $S(\eta)$ constitute net heat flux from the interface.

In the equilibrium state, the heat fluxes in positive y-direction in the fluid phases 1 and 2 are expressed as $-K_1(T_1 - T_0)/h_1$ and $K_2(T_0 - T_2)/h_2$, respectively where K_1 and K_2 are heat conductivities of the two fluids. Let us denote

$$S(y) = \frac{K_2(T_0 - T_2)}{h_2 - y} - \frac{K_1(T_1 - T_0)}{h_1 + y} \tag{8}$$

Expanded $S(\eta)$ in a Taylor series about $\eta = 0$ as

$$S(\eta) = S(0) + \eta S'(0) + \frac{1}{2}\eta^2 S''(0) + \cdots \tag{9}$$

Then we take $S(0) = 0$, so that

$$G = \frac{K_2(T_0 - T_2)}{h_2} = \frac{K_1(T_1 - T_0)}{h_1} \tag{10}$$

This indicates that in the equilibrium state the heat fluxes are equal across the vapor–liquid interface.

Interfacial condition for the conservation of momentum is given by,

$$\begin{aligned}\rho^{(1)}(\nabla\phi^{(1)} \cdot \nabla F)\left(\frac{\partial F}{\partial t} + \nabla\phi^{(1)} \cdot \nabla F\right) &= \rho^{(2)}(\nabla\phi^{(2)} \cdot \nabla F)\left(\frac{\partial F}{\partial t} + \nabla\phi^{(2)} \cdot \nabla F\right) \\ &+ (p_2 - p_1 - 2\mu^{(2)}\boldsymbol{n} \cdot \nabla \otimes \nabla\phi^{(2)} \cdot \boldsymbol{n} + 2\mu^{(1)}\boldsymbol{n} \cdot \nabla \otimes \nabla\phi^{(1)} \cdot \boldsymbol{n} + \sigma\nabla \cdot \boldsymbol{n})|\nabla F|^2\end{aligned} \tag{11}$$

where p represents the pressure, σ denotes the surface tension coefficient and $\mathbf{n}$ is the unit normal vector at the interface, respectively. Surface tension has been assumed to be a constant, neglecting its dependence on temperature.

3. Pressure correction for potential flow analysis

Wang et al. [11] derived a viscous correction for the irrotational pressure at the free surfaces of steady flows, to resolve the discontinuity between the non-zero shear stress and zero shear stress condition at the free surfaces. We will derive the pressure correction for Kelvin–Helmholtz instability from the basic mechanical energy equation.

Suppose that $\boldsymbol{n}_1 = \boldsymbol{e}_y$ is the unit outward normal at the interface for the inside fluid; $\boldsymbol{n}_2 = -\boldsymbol{n}_1$ is the unit outward normal for the outside fluid; $\boldsymbol{t} = \boldsymbol{e}_x$ is the unit tangent vector. We use "i" for

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