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Scaling for the Prandtl number of the natural convection boundary layer of an inclined flat plate under uniform surface heat flux

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ABSTRACT

An improved scaling analysis and direct numerical simulations are performed for the unsteady natural convection boundary layer adjacent to a downward facing inclined plate with uniform heat flux. The development of the thermal or viscous boundary layers may be classified into three distinct stages: a start-up stage, a transitional stage and a steady stage, which can be clearly identified in the analytical as well as the numerical results. Previous scaling shows that the existing scaling laws of the boundary layer thickness, velocity and steady state time scale for the natural convection flow on a heated plate of uniform heat flux provide a very poor prediction of the Prandtl number dependency of the flow. However, those scalings perform very well with Rayleigh number and aspect ratio dependency. In this study, a modified Prandtl number scaling is developed using a triple-layer integral approach for $Pr > 1$. It is seen that in comparison to the direct numerical simulations, the modified scaling performs considerably better than the previous scaling.

1. Introduction

The free convection heat transfer of the boundary layer adjacent to a vertical or inclined flat plate is a common phenomenon in nature and in industry. The present study is of practical significance in both fluid mechanics and heat transfer research communities. Natural convection heat transfer through an inclined surface is frequently encountered in nature and in engineering devices such as solar water heaters and attic roof spaces. In particular, an increasing number of studies have focused on natural convection adjacent to an inclined semi-infinite flat plate [1–5]. However, most of the studies have been conducted by either numerical simulations or experimental observations. Few theoretical studies have also been performed for this kind of problems.

Mathematical analysis, called scaling analysis, of the transient behavior of the flow in the boundary layer has been considered by many researchers recently. It is a cost-effective way that can be applied for understanding the physical mechanism of the fluid flow and heat transfer. The results of scaling analysis also play an important role in guiding both further experimental and numerical investigations. Patterson and Imberger [6] conducted the scaling analysis on the transient behavior of the flow of a differentially heated cavity. The authors classified the flow development through several transient flow regimes into one of three steady-state types of flow based on the relative values of the Rayleigh number Ra, the Prandtl number Pr, and the aspect ratio A. Scaling has become popular since then. A considerable number of research have been conducted for many aspects of unsteady natural convection boundary layer flow under various flow configurations through scaling analysis, some of which have been verified through comparisons with direct numerical simulations over a range of forcing parameters [7–12].

Scaling analysis has also been performed for various thermal forcing conditions, e.g. sudden and ramp temperature variations [14–17], surface heating due to radiation [18], uniform surface heat flux [19–22], etc. The scaling analysis of the boundary layer under the inclined walls of an attic space for both heating and cooling roof conditions has been performed recently (see [23–25]). However, Poulikakos and Bejan [26] first conducted scaling analysis for this geometry by considering the situation for a very small roof slope with Prandtl number greater than unity. It is worth noting that thermal and viscous boundary layers, whose thicknesses increase with time to constant values at steady state, developed under both roof planes. Saha et al. [24,25] and Saha [23,27] revisited the attic space problem for both $Pr \geqslant 1$ and several thermal forcing conditions for a wide range of roof slope. The authors also developed the heating-up and cooling-down time scales for the entire enclosure and the transient heat transfer scales as a form of Nusselt number. The derived scales have been verified by the numerical simulations for a range of aspect ratios, Prandtl numbers and Rayleigh numbers.

In most of the above studies, the existing scaling relations do not provide good prediction of the Prandtl number dependency of the flow for the velocity field. Recently, a modification of the

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Nomenclature

A	slope of the plate
L	length of the plate
l	length of the horizontal projection of the plate
g	acceleration due to gravity
h	length of the vertical projection of the plate
P	dimensional pressure
p	dimensionless pressure
Pr	Prandtl number
Ra	Rayleigh number
t	dimensional time
t_s	dimensional steady state time
T	dimensional temperature of the fluid
T_w	dimensional temperature scale on the plate
u, v	dimensionless fluid velocities in the x- and y-direction respectively
U_0	reference velocity
U, V	dimensional fluid velocities in the X- and Y-direction respectively
U_m	dimensional maximum velocity
U_{ms}	dimensional velocity scale at steady state stage
u_m	dimensionless maximum velocity
u_{ms}	dimensionless velocity scale at steady state stage
x, y	dimensionless Cartesian coordinates
X, Y	dimensional Cartesian coordinates

Greek letters

β	thermal expansion coefficient
ΔT	temperature difference
$\delta_T - \delta_i$	dimensional viscous inner layer thickness
$\delta_{Ts} - \delta_{is}$	dimensional steady state viscous inner layer thickness
$\Delta_T - \Delta_i$	dimensionless viscous inner layer thickness
$\Delta_{Ts} - \Delta_{is}$	dimensionless steady state viscous inner layer thickness
δ_T	dimensional thermal layer thickness
δ_{Ts}	dimensional steady state thermal layer thickness
Δ_T	dimensionless thermal layer thickness
Δ_{Ts}	dimensionless steady state thermal layer thickness
δ_v	dimensional viscous layer thickness
δ_{vs}	dimensional steady state viscous layer thickness
Δ_v	dimensionless viscous layer thickness
Δ_{vs}	dimensionless steady state viscous layer thickness
Γ_w	heat flux
κ	thermal diffusivity
ρ	density of the fluid
ν	kinematic viscosity
θ	dimensionless temperature
θ_w	dimensionless temperature scale on the plate
ϕ	angle
τ	dimensionless time
τ_s	dimensionless steady state time

scaling has been performed for both sudden [10,15,16,23] and ramp heating [13,14,27] boundary conditions. The modified scaling relations describing the Prandtl number dependency agree very well with the direct numerical simulation results for a wide range of Pr values following $Pr > 1$. However, modified scaling for the heat flux case has not been performed for an inclined flat plate. This is the motivation for the present study.

In this study, a three-region scaling analysis for the development of the boundary layers adjacent to a downward facing inclined heated flat plate is performed for uniform heat flux conditions. The Prandtl number chosen in this study is greater than unity. Detailed balances of the important terms of the Navier–Stokes and the energy equations are examined. The scaling relations of the velocity, thermal and viscous layer thicknesses in the different stages of the boundary layer development are achieved, and the time scale of the transition of the flow to a steady state is obtained, as is the time scale. A number of numerical simulations are performed for different flow parameters: Rayleigh number (Ra), Prandtl number (Pr) and slope of the plate (A) in order to validate these scaling relations. It is found that the numerical results agree well with the scaling results for all parameters considered in this study.

2. Problem formulation

Under consideration is the flow resulting from an initially motionless and isothermal Newtonian fluid with $Pr > 1$ adjacent to a downward facing inclined heated plate due to uniform heat flux. The physical system shown in Fig. 1 consists of an inclined flat plate of heated length L. We extend both ends of the plate by a distance equal to its length at the right end and half the length at the left end to form a rectangular domain, which is filled with an initially stationary fluid at a temperature T_0. If we consider the plate as the hypotenuse of a right angled triangle then the height is h, the length of the base is l and the angle that the plate makes with the base is ϕ. Except for the heated length L (shown in Fig. 1), all the boundaries of the rectangular domain are assumed to be adiabatic, rigid and non-slip. A uniform surface heat flux is applied to the plate.

The development of the flow under the inclined plate is governed by the following two-dimensional Navier–Stokes and energy equations with the Boussinesq approximation:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \tag{1}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{1}{\rho}\frac{\partial P}{\partial X}+\nu\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)+g\beta\sin\phi T, \tag{2}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{1}{\rho}\frac{\partial P}{\partial Y}+\nu\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+g\beta\cos\phi T, \tag{3}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\kappa\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right). \tag{4}$$

Initially the fluid is quiescent and isothermal. All boundaries are assumed to be non-slip. Except for the plate, the adiabatic condition is also assumed for the temperature. On the plate the temperature condition is defined as

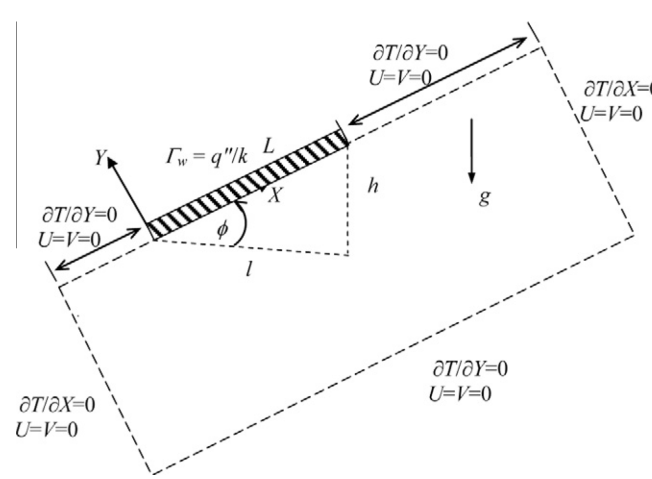

Fig. 1. Schematic of the computational domain and boundary conditions.