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Mathematical optimisation of laminar forced convection heat transfer through a vascularised solid with square channels

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ABSTRACT

This paper presents a three-dimensional geometric optimisation of cooling channels in forced convection of a vascularised material with the localised self-cooling property subjected to a heat flux. A square configuration was studied with different porosities. Analytical and numerical solutions were provided. The geometrical configuration was optimised in such a way that the peak temperature was minimised at every point in the solid body. The optimisation was subject to the constraint of a fixed global volume of solid material, but the elemental volume was allowed to morph. The solid material was subject to a heat flux on one side and the cooling fluid was forced through the channels from the opposite direction with a specified pressure difference. The structure had three degrees of freedom as design variables: the elemental volume, channel hydraulic diameter and channel-to-channel spacing. A gradient-based optimisation algorithm was used to determine the optimal geometry that gave the lowest thermal resistance. This optimiser adequately handled the numerical objective function obtained from numerical simulations of the fluid flow and heat transfer. The numerical results obtained were in agreement with a theoretical formulation using scale analysis and the method of intersection of asymptotes. The results obtained show that as the pressure difference increases, the minimised thermal resistance decreases. The results also show the behaviour of the applied pressure difference on the optimised geometry. The use of the optimiser made the numerical results to be more robust with respect to the optimum internal configurations of the flow systems and the dimensionless pressure difference.

1. Introduction

Material with the property of self-healing and self-cooling is becoming more promising in heat transfer analysis [1–7]. The development of vascularisation of materials indicates flow architectures that conduct and circulate fluids at every point within the solid body. This solid body (slab) may be performing or experiencing mechanical functions such as mechanical loading, sensing and morphing. The self-cooling ability of vascularised material to bathe volumetrically at every point of a solid body gave birth to the name "smart material". Constructal theory and design [8,9] have been adopted as an optimisation technique for the development of a procedure that is sufficiently allocating and optimising a fixed global space constraint using a physical law (constructal law). The method seeks to optimise the flow architecture that predicts the flow and thermal fluid behaviour in a structure that is subject to a global volume constraint. Bejan [8,9] stated this law as: *For a finite-size system to persist in time (to live), it must evolve in such a way that it provides easier access to the imposed (global) currents that flow through it*. In a smart material, constructal theory ideally helps in the vascularisation of the smart material structure by morphing the flow architecture configuration to provide easier and greater access of flow through it.

The application of this theory started with Bejan and Sciubba [10], who obtained a dimensionless pressure difference number for optimal spacing from board to board of an array of parallel plates to channel length ratio and a maximum heat transfer density, which can be fitted in a fixed volume in an electronic cooling application using the method of intersection of asymptotes. The applications of this theory have been reviewed [11,12] where, under certain global constraints, the best architecture of a flow system can be achieved with the one that gives less global flow resistances, or allows high global flow access. In other words, the shapes of the channels and the elemental structure that is subject to global constraint are allowed to morph. The optimisation of heat exchangers and multiscale devices by constructal theory has also recently been reviewed and summarised by Reis [13] and Fan and Luo [14].

Bello-Ochende et al. [15] conducted a three-dimensional optimisation of heat sinks and cooling channels with heat flux using

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Nomenclature

A_c	cross-sectional area of the channel, m^2
A_s	cross-sectional area of the structure, m^2
Be	Bejan number
C_P	specific heat at constant pressure, J/kg K
d_h	hydraulic diameter, m
H	structure height, m
h	elemental height , m
h_c	channel height , m
i	mesh iteration index
k	thermal conductivity, W/mK
L	axial length, m
N	number of channels
n	normal
P	pressure, Pa
q''	heat flux, W/m^2
R	thermal resistance
s	channel spacing, m
T	temperature, °C
u	velocity, m/s
$\vec{u}$	velocity vector, m/s
V	global structure volume, m^3
v_c	channel volume, m^3
v_{el}	elemental volume, m^3
W	structure width, m
w	elemental width, m
w_c	cooling channel width, m
x, y, z	Cartesian coordinates, m

Greek symbols

α	thermal diffusivity, m^2/s
μ	viscosity, kg/m s
ρ	density, kg/m^3
∂	differential
∞	far extreme end, free stream
ϕ	porosity
Δ	difference
∇	differential operator
γ	convergence criterion

Subscripts

f	fluid
in	inlet
max	maximum, peak
min	minimum
opt	optimum
r	ratio
s	solid
w	wall

scale analysis and the intersection of asymptotes method based on constructal theory to investigate and predict the design and optimisation of the geometric configurations of the cooling channels. Rocha et al. [16] and Biserni et al. [17] applied the theory to optimise the geometry of C- and H-shaped cavities respectively that intrude into a solid conducting wall in order to minimise the thermal resistance between the solid and the cavities, Lorenzini et al. [18] used the theory to minimise the thermal resistance between the solid and the cavities by optimising the geometry of isothermal cavities that evolve from T- and Y-shaped of a solid conducting wall.

Cho et al. [19] numerically investigated the flow and thermal behaviour of vascular cooling plate for the volumetric bathing of the smart structures. Constructal theory applications on the vascularisation revolution of smart materials can also be found in open literature [20–25]. Also, the constructal theory for optimisation of several components and systems in engineering applications has been extensively discussed and documented in the literature [26–29].

The recent comment by Meyer [30] on the latest review of constructal theory by Bejan and Lorente [31] shows that the application of constructal law in all fields of educational design is a wide road to future advances.

This paper is borne out of the work of Kim et al. [7], who theoretically and numerically analysed vascularised materials with heating from one side and coolant forced from the other side for parallel plates and cylindrical channel configurations in an attempt to find the channel configurations that minimised the non-uniform temperature distribution of a vascularised solid body. This paper focuses on the mathematical optimisation of laminar forced convection heat transfer through a vascularised solid with square channels. It examines the optimisation of a fixed and finite global volume of solid material with an array of square cooling channels, with a uniform heat flux from one side. The objective is the building of a smaller construct to form part of a larger construct body with a self-cooling function that will lead to the minimisation of the global thermal resistance or, inversely, the maximisation of the heat transfer rate density (the total heat transfer rate per unit volume). This is achieved by designing the body in a vascularised manner and by forcing a coolant to the heated spot in a fast and efficient way so as to significantly reduce the peak temperature at any point inside the volume that needs cooling. The solution of Kim et al. [7] will be used as comparison for the results reported in this paper.

2. Computational model

The schematic diagram of the physical configuration is shown in Fig. 1. The system consists of a solid body of fixed global volume, V, which is heated with uniform heat flux q'' on the left side. The body is cooled by forcing a single-phase cooling fluid (water) from the right side through the parallel cooling channels. The flow is driven along the length L, of the square channel ($w_c = h_c$) with a fixed pressure difference ΔP, in a transverse and counter-direction to the heat flux. An elemental volume shown in Fig. 2 consisting of a cooling channel and the surrounding solid was used for analysis because of the assumption of the symmetrical heat distribution on the left side of the structure. The heat transfer in the elemental volume is a conjugate problem, which combines heat conduction in the solid and the convection in the working fluid.

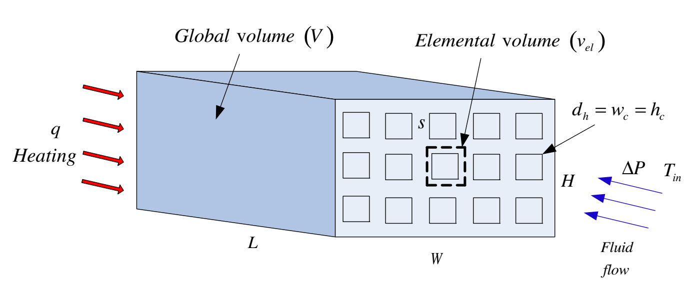

Fig. 1. Three-dimensional parallel square channels across a slab with heat flux from one side and forced flow from the opposite side.