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Dual solutions in a double-diffusive convection near stagnation point region over a stretching vertical surface

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ABSTRACT

The development of double-diffusive convection near stagnation point region over a stretching vertical surface with constant wall temperature has been investigated. The external flow and the stretching velocities are assumed to vary with $\sqrt{x}$, where x is the distance from the slot where the stretching surface is issued. Using the local similarity method, it has been shown that a set of suitable similarity transformations reduces the non-linear coupled partial differential equations governing the flow, thermal and concentration fields into a set of non-linear coupled ordinary differential equations. The non-linear self-similar equations along with the boundary conditions form a two point boundary value problem and are solved using Shooting method, by converting into an initial value problem. In this method, the system of equations is converted into the set of first order system which is solved by fourth-order Runge–Kutta method. Flows with both assisting and opposing buoyancy forces are considered in the present investigation. The study reveals that the dual solutions of velocity, temperature and concentration exist for certain values of suction/injection and buoyancy parameters. Prandtl and Schmidt numbers strongly affect the thermal and concentration boundary layer thicknesses, respectively. The effects of various parameters on the velocity, temperature and concentration profiles are also presented here.

1. Introduction

Free convection is caused by the temperature difference of the fluid at different locations and forced convection is the flow of heat due to the cause of some external applied forces. The combination of free convection and forced convection is called as mixed convection. The phenomenon of mixed convection flow near a stagnation point has attracted several investigators during the recent past because of its wide range of applications in many industrial problems. In such flows, heat transfer in the boundary layer adjacent to continuous moving surfaces has also various important applications in manufacturing processes. Examples may be found in continuous casting, glass fiber production, metal extrusion, hot rolling, the cooling and/or drying of paper and textiles and wire drawing.

Among the earlier studies reported in the literature, Ridha [1], Merkin and Mahmood [2], and Wilks and Bramley [3] have discussed the dual similarity solutions in the context of mixed convection boundary layer flow over an impermeable vertical flat plate. In such cases, the mixed convection flows are characterized by the buoyancy parameter λ which depends on the flow configuration and the surface heating conditions. Parameter λ provides a measure of the influence of the free convection on comparison with that of forced convection in the fluid flow. Assisting or aiding flows are flows for which the buoyancy force has a positive component in the direction of the free stream velocity, and those flows for which the buoyancy force has a component opposite to the free stream velocity will be designated as opposing flows. The existence of non-unique (dual) similarity solutions in mixed convection boundary layer flow for the opposing flow case was reported by several researchers, see for example, [2–7]. It may be remarked that Ridha [1] probably is the first to show that dual solutions exist in the opposing flow regime and they continue into the assisting flow regime. It may be noticed that Nazar et al. [8] have re-investigated the problem posed by Merkin and Mahmood [2], and reported the existence of dual solutions only in the case of opposing flow. Merril et al. [9] improved the result obtained by Nazar et al. [8] showing that the lower branch solutions exist in the opposing flow regime and they continue into the assisting flow regime. Also, Merril et al. [9] found that dual solutions existed for all values of the buoyancy parameter $\lambda \succ \lambda_c$, where $\lambda_c(\prec 0)$ was the value of λ for which the upper branch solution met the lower branch solution.

It may also be noted that, on a different approach, Merril et al. [9] have performed a stability analysis for different steady state solutions of a mixed convection flow on a vertical surface near the stagnation point. They have reported the existence of dual

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Nomenclature

a, b	constants
C	fluid concentration
C_{fx}	local skin friction coefficient
D	mass diffusivity
f	dimensionless streamfunction
F	dimensionless velocity
g	acceleration due to gravity
Gr_x, Gr_x^*	local Grashof numbers
k	thermal conductivity
N	ratio of buoyancy parameters
Nu_x	local Nusselt number
Pr	Prandtl number, Pr = ν/α
Re_x	local Reynolds number based on x, $\mathrm{Re}_x = U_w(x)x/\nu$
Sc	Schmidt number, Sc = ν/D
Sh_x	local Sherwood number
T	fluid temperature in the boundary layer
u, v	velocity components in the x - and y-directions, respectively
$u_e(x)$	velocity of the external flow
$u_w(x)$	velocity of the stretching surface
x, y	Cartesian coordinates measured along the surface and normal to it, respectively

Greek symbols

α	thermal diffusivity
β	volumetric coefficient of thermal expansion
β^*	volumetric coefficient of expansion for concentration
$\varepsilon(= a/b)$	velocity ratio parameter
η	similarity variable
θ	dimensionless temperature
λ, λ^*	buoyancy parameters
μ	dynamic viscosity
ν	kinematic viscosity
ρ	fluid density
ϕ	dimensionless concentration
ψ	dimensional streamfunction

Subscripts

e	condition at the edge of the boundary layer
w	condition at the wall
∞	freestream condition

solutions where the upper branch solutions are linearly stable while those on the lower branch are linearly unstable. Another point worthy to mention that Ishak et al. [10–12] and Ridha [13] have also reported in their respective studies that the lower branch solutions are deprived of physical significance and either such solution may remain of mathematical interest in some situations or similar equations may reappear in other situations which could have more realistic meaning [10–13]. On the other hand, Chiam [14] studied the stagnation point flow over a stretching sheet where he considered identical stretching velocity and straining velocity and found no boundary layer structure near the sheet. However, a few important investigations regarding the stagnation point flow over stretching sheet were made by Nadeem et al. [15,16], Layek et al. [17], Ishak et al. [18] and Mahapatra and Gupta [19]. Also, there are some recent studies on flow over a shrinking sheet as reported by Nadeem et al. [20–22]. For many practical applications, the continuous moving surface undergoes many processes like stretching and cooling or heating that cause surface velocity and temperature variations. In fact, Chen [23] has shown the effects of thermal buoyancy on flow over a vertical, continuous stretching sheet where the velocity and temperature distributions are assumed to vary according to a power-law form. Later, Ishak and Nazar [24] have discussed the steady magneto-hydrodynamic (MHD) mixed convection boundary layer flow of an incompressible viscous and electrically conductive fluid over a stretching vertical surface with constant wall temperature in presence of an applied magnetic field.

It has been noticed that earlier investigators have not reported the effect of mass diffusion in the literature whereas in many real world problems the flow, the heat transfer and the mass diffusion are always coupled. In fact, the buoyancy forces arising from the simultaneous effects of temperature and concentration differences play a significant role in mixed convective thermal and concentration double diffusion when the flow velocity is relatively small and the temperature difference or concentration difference is relatively large. Double-diffusive convection is formed due to a combination of temperature and concentration gradient in the fluid, in which the thermal and mass diffusivities are different. Thus, the heat and mass transfer occur simultaneously. In practice, double-diffusive convection may appear in a wide range of scientific fields such as Biology, Oceanology, Astrophysics, Geology, Chemical processes and Crystal-growth techniques. Therefore in an attempt to the subsequent development in literature, authors are motivated to investigate the simultaneous effects of thermal and mass diffusions on a mixed convection flow over a stretching vertical surface.

The aim of the present paper is to study the double diffusive mixed convection flow with suction or injection near the stagnation point region over a stretching vertical surface. The self-similar solution of the coupled non-linear partial differential equations governing the mixed convective flow has been obtained numerically using Shooting method to represent dual solutions. The results for some particular cases are compared with those of Chen [23] and Ishak et al. [24].

2. Analysis

A steady two dimensional laminar mixed convection flow near stagnation point region over a stretching vertical surface in a viscous fluid of temperature T_∞ and concentration C_∞ (see Fig. 1) is considered. Either heating or cooling of the plate is assumed to begin simultaneously with the motion of the external stream. The external velocity is prescribed as $u_e(x) = a\sqrt{x}$ and the stretching velocity is assumed to be of the form $u_w(x) = b\sqrt{x}$, where a and b are constants with $a \geqslant 0$ and $b \succ 0$. T_w and C_w are temperature and concentration at the wall assumed to be constant. The buoyancy forces arise due to the variations in temperature and concentration of fluid. The Boussinesq approximation is invoked for the fluid properties to relate the density changes to temperature and concentration and to couple in this way the temperature and concentration fields to the flow field. Under these assumptions, the governing boundary layer equations can be expressed as [25]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$