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A theoretical study on drop breakup modeling in turbulent flows: The inertial subrange versus the entire spectrum of isotropic turbulence

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HIGHLIGHTS

- A study of breakup models in the wide spectrum of turbulence.
- The different turbulence models influence significantly on the breakup model prediction.
- Inconsistencies of previous breakup models have been elucidated.

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ABSTRACT

The traditional model framework for drop breakup in turbulent flows is based on the inertial subrange of turbulence. That is, Kolmogorov's formulas for the energy spectrum and second-order longitudinal structure function are used. In recent literature the model framework has been extended to consider the wide energy spectrum (i.e. including the dissipation, inertial and energy-containing subranges of turbulence). In particular, two different formulas have recently been proposed for the second-order longitudinal structure function based on the wide energy spectrum. The comparison between these two formulas reveals significantly different predictions of the breakup phenomenon for particular conditions.

It is important to use the Pope model energy spectrum (valid for the wide spectrum of turbulence) consistently (Pope, S.B., 2000. Turbulent Flows. Cambridge University Press, Cambridge). That is, parameter fitting must be performed on the parameters of the energy spectrum function when the physical conditions of the system is changed. Although the parameter values given in the original literature by Pope are valid only at sufficiently high Reynolds number, these parameter values have been employed at low Reynolds numbers by some researchers. With decreasing Reynolds numbers the difference between employing the original suggested values and re-fitted parameter values in models for breakage is increasingly significant.

In the development of new models for the daughter size distribution function, the number and volume conservation properties should always be analyzed. Care should be taken when a change in the model parameter is performed, for example, the Jacobian relation in an integral is required for consistency. Precise notation regarding the function definitions is required in order to avoid model misinterpretations.

1. Introduction

As turbulence significantly enhances heat and mass transfer rates, the majority of flows encountered in industrial applications are operated in turbulent regimes. For example, turbulence plays an important role in dispersed multiphase flows because it affects processes such as breakup and coalescence of drops and bubbles. The breakup and coalescence phenomena directly influence on the interfacial area between the continuous and dispersed phases (Andersson et al., 2015).

Population balances can be used to describe changes in the fluid particle size distribution, and hence the interfacial area between the continuous and dispersed phases (e.g. Ramkrishna, 2000; Yeoh et al., 2014). The main weakness in utilizing a population balance equation relates to the underlying models for breakup and coalescence. For example, different criteria for breakup are employed developing the various breakup models, hence the prediction of the breakup events may differ significantly

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Notation

Upper-case Latin letters

C	Kolmogorov parameter of the energy spectrum
$\mathcal{C}$	Kolmogorov parameter of the structure function
$C_{t,3}$	model parameter of the ternary breakup model
$C_{t,4}$	model parameter of the quaternary breakup model
C_L	parameter of the Pope (2000) model energy spectrum
C_η	parameter of the Pope (2000) model energy spectrum
E	energy spectrum, m^3/s^2
F	hypergeometric function
K	Bessel function
L	integral scale, m
N	numerical resolution/number of discretization points
P_n	breakup probability
P_0	parameter of the Pope (2000) model energy spectrum
Re_L	integral scale Reynolds number
S_0	dimensionless oscillation ratio
V_0	volume of the mother drop, m^3
V	volume, m^3

Lower-case Latin letters

b_n	breakage frequency, s^{-1}
$c_{f,n}$	constraint of surface energy increase
$c_{d,n}$	constraint of energy density increase
c_0	parameter in second-order structure function (14), defined by (15)
d_0	diameter of the mother drop, m
$e(\lambda)$	kinetic energy of an eddy of size λ, J
f_d	diameter fraction (d/d_0)
f_v	volume fraction (V/V_0)
k	kinetic energy, $m^2\,s^{-2}$
n	number of daughter drops (2, 3, 4)
n_λ	number density of eddies of size λ, $m^{-3}\,m^{-1}$
n_{d_0}	number density of dispersed mother drops, m^{-3}
n_κ	number density of eddies of wave-number κ, $m^{-3}\,m^{-1}$
$\tilde{n}_\kappa$	"number density" of eddies of wave-number κ, m^{-2}
r	distance between two velocity points, m
$\bar{u}$	mean velocity in a turbulent eddy, m/s
$\bar{u}_\kappa$	mean velocity of eddy of size κ, m/s
$\bar{u}_\lambda$	mean velocity of eddy of size λ, m/s
x	auxiliary variable (λ/d_0)
z	auxiliary variable (κd_0)

Greek letters

α_d	dispersed phase volume fraction
β	parameter of the Pope (2000) model energy spectrum
β_n^v	volume-based daughter size distribution function
β_n^d	diameter-based daughter size distribution function
$\chi_{c,n}$	critical dimensionless energy
η	Kolmogorov microscale, m
Γ	gamma function
κ	wave-number, 1/m
λ	eddy size, m
μ_c	dynamic viscosity of continuous phase, kg/(m s)
ν	kinematic viscosity, m^2/s
ν_c	kinematic viscosity of continuous phase, m^2/s
ω	frequency density, $m^{-3}m^{-1}\,s^{-1}$
$\tilde{\omega}$	"frequency density", $m^{-2}\,s^{-1}$
Ω_n^v, Ω_n^d	volume and diameter based breakage frequency densities, $m^{-3}\,s^{-1}$
φ_n	partial breakage density, $m^{-3}s^{-1}$
ψ_n	breakage rate of drops of volume V_0 into n daughter drops of volumes $f_{v,1}V_0, \ldots, f_{v,n}V_0$, $m^{-3}\,s^{-1}$
ρ_c	density of continuous phase, kg/m^3
ρ_d	density of dispersed phase, kg/m^3
σ	interfacial tension, N/m
τ_e	eddy turnover time or lifetime, s
ε	energy dissipation rate, m^2/s^3

Other symbols

$\langle[\delta v]^2\rangle$	one-dimensional second-order longitudinal structure function, m^2/s^2

Abbreviation

HIST	Han et al. (2011, 2013) breakage model in the inertial subrange of turbulence
HEST	Han et al. (2014, 2015) breakage model in the entire spectrum of turbulence
SEST	Solsvik and Jakobsen (2016a) breakage model in the entire spectrum of turbulence

between the proposed breakup models in the literature. Similar limitations in the model framework also exist for the coalescence phenomenon (see e.g. reviews by Liao and Lucas, 2009, 2010). Thus, both fundamental experimental investigations and modeling studies are continued to improve the understanding and description of the mechanisms for fluid particle breakup and coalescence in order to further develop the existing model framework (e.g. the recent work by Becker et al., 2014; Ghasempour et al., 2014b; Han et al., 2015; Maaß et al., 2011; Maaß and Kraume, 2012; Nachtigall et al., 2016; Orvalho et al., 2015; Solsvik and Jakobsen, 2015; Solsvik et al., 2015b; Villwock et al., 2014).

The standard model framework for fluid particle breakup and coalescence in turbulent flows is limited to the inertial subrange of turbulence (see e.g. Coulaloglou and Tavlarides, 1977; Luo and Svendsen, 1996; Prince and Blanch, 1990). In recent literature attempts have been made to extend the model framework to consider the entire spectrum of turbulence, which consists of the energy-containing, inertial, and dissipation subranges (e.g. Ghasempour, 2015; Ghasempour et al., 2014a,b; Han et al., 2014, 2015; Solsvik and Jakobsen, 2016a). A review of the statistical turbulence theory is provided by Solsvik and Jakobsen (2016b).

Han et al. (2014) extended their binary breakup model valid for the inertial subrange of turbulence (i.e. Han et al., 2011) to the entire spectrum of turbulence. This model extension was obtained by replacing Kolmogorov's formulas for the energy spectrum function and second-order structure function (Kolmogorov, 1941a, b) with the energy spectrum function suggested by Pope (2000) and based on the work by Lamont and Scott (1970) the authors proposed a new semi-empirical relation for the second-order structure function. Later, Han et al. (2015) also extended their model framework for multiple breakup in the inertial subrange of turbulence (i.e. Han et al., 2013) to the entire spectrum of turbulence.

A new semi-empirical relation for the second-order structure function for the entire spectrum of turbulence was recently proposed by Solsvik and Jakobsen (2016a). The new formula showed