International Journal of Heat and Mass Transfer 55 (2012) 2676–2683

Contents lists available at SciVerse ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Approximation accuracy of the two-point third moments of the velocity field in the homogeneous turbulence

V.A. Babenko [a,*], V.A. Frost [b]

[a] Institute of Heat and Mass Transfer, NAS, Minsk, Belarus
[b] Institute for Problems in Mechanics, RAS, Moscow, Russia

ARTICLE INFO

Article history:
Received 23 November 2011
Accepted 5 December 2011
Available online 16 January 2012

Keywords:
Turbulence
Correlation functions
Karman–Howart equation

ABSTRACT

Different approximations for two-point third moments of velocity field appearing in the Karman–Howart equation are compared. For this purpose, the known experimental results of Townsend and Stewart and the model form of turbulence energy spectrum, which enables approximating the experimentally determined second moments of velocity field, are used. The latter are applied for evaluation of the two-point third moments by the procedures proposed by various authors. Calculation results are compared to experimental data, which allows obtaining quantitative assessments of approximation accuracy. For a number of models, the second-order structural function is found from the Kolmogorov equation for inertial subrange. Thus, in the Hasselman and Lytkin models for the structural function D_{LL} the power law expected in the inertial subrange $D_{LL}(r) \sim r^{2/3}$ is obtained.

1. Introduction

In the case of turbulent non-premixed combustion, the chemical reaction proceeds basically in zones where flows appear well premixed, and the required components meet randomly in necessary ratios. Therefore, for flows with comparable reference time scales of chemical reaction and of turbulent micromixing the strong spatial heterogeneity of reaction layers is peculiar. At the same time, the majority of the theoretical models of micromixing is one-point and, by virtue, cannot consistently take into account the structure of turbulent fields.

The modern statistical models of turbulent combustion – flamelet, conditional moment closure, transfer of a joint probability density function (PDF) of scalars – take into consideration the heterogeneity with the use of some *a priori* assigned characteristics of the turbulent flow spatial structure such as a conditional rate of scalar dissipation, a conditional rate of diffusive transfer, reference scales of length and time of a scalar field and so on. The necessity to improve these characteristics is admitted by many contributors. Actually, the refinement of the above-mentioned models of turbulent combustion depends on the accumulation of knowledge of these characteristics. For this purpose, experiments and DNS are carried out, and additional models with regard to the turbulence spatial structure are developed as well.

Such additional models can be formulated for correlation or structural functions of velocity and scalar fields [1–7], for two-point PDFs, and also for a joint PDF of scalars and their gradients, or scalars and scalar dissipation.

The experimental study of turbulence spatial structure is followed in measurement of spectra and correlation functions. In most cases, primary data of experiments represent two-point or two-time correlations or structural functions. It is therefore of importance to develop theoretical models for correlation functions of velocity and scalar fields that are adequate to experiment and DNS.

As contrast to the probability density function (PDF), the correlation function is a more convenient mathematical object as being smooth function with a regular behavior. However, as well as in the case of the PDF usage, the amount of independent variables increases, thus essentially complicating the solution process. Only in the most elementary cases, such as a flat mixing layer or a spherically symmetric cloud, it is possible considering one space variable. The total number of variables in these cases corresponds to 3D geometry.

One more difficulty is that the equations for the CF are unclosed as they contain the two-point third moments of velocity field which cannot be immediately expressed through the CF. Different closure methods for these equations are known [8,9], and there appears a problem of choosing the most justified model corresponding to physical representations and not introducing too considerable numerical difficulties.

Our task is the examination of the methods grounded on different assumptions permitting to express the third moments, appearing in the equation for CF, through a required correlation function. As a major similarity exists in the description of correlation

* Corresponding author.
E-mail address: bab@hmti.ac.by (V.A. Babenko).

0017-9310/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2011.12.025

functions for the velocity and scalar fields, the model [6] designed for closure of the Corrsin equation is also included in this list.

The experimentally obtained distributions of the third moments [10] are used for estimation of the accuracy of different approximations. As the measure of agreement, the individual variation δM was selected, where $\delta M = \sqrt{\frac{\sum_i (B_{LL,L}(r_i) - B^M_{LL,L}(r_i))^2}{N}}$. Here N is the number of points r_i, at which the value of the two-point third moment $B_{LL,L}$ is determined from experiment, $B^M_{LL,L}$ is the model expression for $B_{LL,L}$defined by a particular closure model.

The testing of a large group of the models [6,2–5,7] has shown a close enough agreement of the results of the models. This has not allowed us to make a definite choice for the benefit of any one. Another way of the choice consists in inspecting calculation based on different approaches with known regularities, for example, with the law of "two thirds" for the second order structural function of velocity in the inertial range. This law being the consequence of Kolmogorov's second similarity hypothesis proves to be true in numerous experimental data obtained in the conditions close to those of stationary isotropic turbulence at large Reynolds numbers [11]. In these conditions, the equation containing the two-point longitudinal structural functions of second and third order (Kolmogorov's equation) [12] is fulfilled. Considering this equation with zero initial condition at $r = 0$, the Cauchy problem is obtained, whose analytical solution is possible only for some of the mentioned above models [2,4,5]. For models [2,4] the solutions of Kolmogorov's equation correspond to " two thirds" power law.

2. Approximation of correlation function

In most cases, the turbulence parameters necessary for approximation of the third moments can be obtained if the one-time two-point CF is known. From the methods considered below, only the approach based on the approximation of direct interaction, see [6], needs a two-time structural function.

In the present paper, for determination of the one-time longitudinal CF of velocity field the turbulence energy spectral distribution assigned parametrically in the form of several known regularities for different spectral subranges is used. Namely, the following subranges are considered: large scale subrange (small wave numbers), main subrange with a wide scope of wave numbers, where in some cases the law of "5/3" is observed, and dissipation subrange. The required CF is determined by the relevant integral transformation applied to the spectrum (1). The positive definiteness of a spectrum ensures a statistical reliability of the CF, and the spectrum parameters are adjusted to obtaining the correspondence of the calculated and measured CFs. The measurement data [10] were used for the two-point velocity field correlation functions of second and third order at different distances behind a turbulizing grid.

2.1. Parametric representation of spectrum

Parametric representation of a turbulence energy spectrum was set as the sequence of subranges with known regularities, with continuity matching. In so doing, two forms of the spectrum were used.

The first one contained only three reference subranges of wave numbers: large-scale ($E(k) \sim k^4$), main ($E(k) \sim k^p$) and dissipation ($E(k) \sim exp(-\eta^2 k^2)$). This spectral representation has five unknown parameters: spectrum amplitude, reference scale of wave numbers, wave numbers of change from the large-scale subrange to the main one (k_{emax}) and from the main to the dissipation subrange (k_{diss}), and also the exponent p in the power spectral law for the main interval. As a result of adjusting these parameters, the exponent value at the main subrange appears to be close to $-5/3$, which allows us to identify this subrange to the inertial one.

Additionally, the second form of the spectrum included a smooth transition from the large-scale to the main subrange, and, accordingly, one more additional parameter: reference initial wave number of a transitional subrange.

A priori unknowns parameters were taken from a requirement of a maximum accuracy for approximation of the CF that was estimated by the individual variation δB, defined by the relation $\delta B = \sqrt{\frac{\sum_i (B(r_i) - B^E(r_i))^2}{N}}$. Here N is the number of points r_i at which the values $B^E(r_i)$ are measured, and $B(r_i)$ is defined by relation (1):

$$B_{LL}(r) = 2\int_0^\infty \left(\frac{sin(kr)}{(kr)} + 2\frac{cos(kr)}{(kr)^2} - 2\frac{sin(kr)}{(kr)^3}\right)E(k)dk. \quad (1)$$

The spectrum parameters were adjusted by a sequential modification up to the moment of achieving a local extremum with the use of the so-called "ravine" method. Thus, the values of the spectrum amplitude and the wave number reference scale were determined from the normalization requirements of turbulence energy and dissipation rate:

$$\int_0^\infty E(k)dk = 3/2, \quad \int_0^\infty k^2 E(k)dk = 5.$$

In selected dimensionless variables, the Taylor microscale λ is chosen as a reference spatial scale, and the intensity of the longitudinal component of the fluctuational velocity $\langle u'^2 \rangle$ is equal to unity.

The obtained thus value of k_{diss}, identified with the reciprocal of the Kolmogorov scale η, enables calculating the characteristic Reynolds number

$$Re = \frac{\lambda^2}{\sqrt{15}\eta^2}. \quad (2)$$

A similar way of the CF parametric representation was used in [13] for calculation of statistical distributions of a turbulent field in the homogeneous flow behind a turbulizing grid. A difference consists only in the amount of subranges, over which different expressions for a spectral density are set, and in the way of the best selection of parameters. In [13] the dissipation subrange is absent, and the parameters are adjusted to the intensity of turbulence and the integral scale.

Fig. 1 shows a typical correspondence of the calculation results and experimental data for the CF. Here, the curve $(1 - r/\lambda)^2$, approximating the CF at small argument values is also plotted. Fig. 2 represents the comparison of the experimentally determined one-dimensional spectrum with the spectrum $F_1(k_1)$,found from the model energy distribution

$$F_1(k_1) = \int_{k_1}^\infty \frac{E(k)}{k}\left(1 - \frac{k_1^2}{k^2}\right)dk. \quad (3)$$

Similar results of the CF approximation (see Fig. 3) are gained with the usage of the spectra containing a transitional sunrange between initial and main ones (see Fig. 4). In this case, a little bit worse values of individual variations are observed. Table 1 presents the parameters of spectra providing an optimal approximation of the CF at different distances from the turbulizing grid. Note that the variation of some parameters occurs almost at a time with that of the distance to the grid. The variables calculated from experiment [10] are located below the double line in Table 1. Except for the first position ($x/M = 20$), the agreement of the Reynolds numbers obtained when adjusting the best value for the Kolmogorov scale and determined from experimental data can be considered acceptable.