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The fastest capillary penetration of power-law fluids

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HIGHLIGHTS

- This paper studies dynamics of capillary-driven power-law fluids.
- A model for flow in straight tube and Y-shaped tree network is developed.
- The minimum penetration time is found in the optimized structure.
- Unique optimal transport behaviors are analyzed with different power components.

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ABSTRACT

The engineering and modeling of non-Newtonian power-law fluids (i.e., shear thinning and thickening fluids) in porous media has received wide attention in natural systems, oil recovery, and microfluidic devices. In this work, we theoretically explore the dynamics of power-law fluids in the form of capillary flow confined by a straight tube and a Y-shaped tree network, both of which are basic elements of many advanced materials. The straight tube and the tree network are composed of sub-tubes with different radii and lengths. The proposed model reveals that the evolution of the penetration time to the penetration distance is highly dependent on the viscous and capillary effects. If the viscous resistance is high, the flow is slow. If the capillary pressure increases, the flow accelerates. An interesting question is therefore in what optimal structure is the power-law flow fastest, considering different responses of viscous resistance and capillary force to the structural parameters. Based on optimization of the radius and length distribution of sub-tubes, we find the minimum penetration time of the fluids or the fastest flow in both straight tubes and tree networks under size constraints. The unique optimal transport behaviors of power-law fluids, which are different from those of Newtonian fluids, are analyzed in details with different power components.

1. Introduction

Non-Newtonian power-law fluids have found broad applications in natural and engineering fields, including microfluidic devices (Groisman et al., 2003), blood rheology (Fedosov et al., 2011), oil recovery (Pu et al., 2015), capillary breakup (Zimoch et al., 2013), and droplet ejection (Bartolo et al., 2007). The power-law fluid (or the Ostwald–de Waele fluid) in a circular tube is characterized by a modified Hagen–Poiseuille equation (Christop and Middlema, 1965)

$$Q^n = (\pi R^2 u)^n = -\left(\frac{\pi R^3}{3+\frac{1}{n}}\right)^n \frac{R}{2K}\frac{dp}{dx}, \tag{1}$$

where Q is the volume flow rate, n is the power exponent for power-law fluids, u is the flow velocity, R is the tube radius, p is the pressure drop along x-direction, and K is the flow consistency index. Eq. (1) is computed based on the constitutive equation $\tau = k\dot{\gamma}^n$, where τ and $\dot{\gamma}$ are shear stress and strain rate of the fluid, respectively. When $n=1$, the non-Newtonian fluid power-law fluid becomes Newtonian, and Eq. (1) recovers the Hagen–Poiseuille equation in laminar regime. When $n>1$, the power-low fluid is known for shear-thickening or dilatant as viscosity increases with shear rate. When $n<1$, shear thinning or pseudo-plastic behavior occurs due to the decrease in viscosity at higher shear rates. The more generalized model for the non-Newtonian fluid is described by the Herschel–Bulkley model, $\tau = \tau_0 + k\dot{\gamma}^n$, where τ_0 is the yield stress (Yun et al., 2010). The Herschel–Bulkley model represents the Bingham plastic fluid with $n=1$, and becomes the power-law fluid model with $\tau_0 = 0$. Based on the fractal geometry of porous media (Cai et al., 2010), the flow behavior of power-low fluids was quantified by fractal dimensions from single tortuous tubes (Yun et al., 2008) to generous porous

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systems with wide pore size distribution (Zhang et al., 2006). Eq. (1) was also extended to model the power-law flow through packed and porous media by introducing the Blake–Kozeny equation as the permeability (Christop and Middlema, 1965). Without using empirical constants, a resistance model of power-law flow in packed beds was recently developed by taking into account the inertia force (Tang and Lu, 2014). The power-law fluid through three-dimensional disordered porous media was explored numerically with a broad range of Reynolds number, and an enhancement of permeability was found at intermediate conditions due to the interplay of fluid rheology, disordered geometry, and inertial effects (Morais et al., 2009).

The self-driven power-law fluids caused by the capillary force have unique transport properties. The capillary dynamics of the power-law fluids in a uniform circular tube was expressed as a function of n, and the flow was found to be retarded when the liquid became more strongly shear-thinning with $n<1$ (Turian and Murad, 2005). The impact of fluid rheology and dynamic contact angle on the capillary rise of power-law fluid was explored experimentally and theoretically (Digilov, 2008). The shear-thinning fluid under gravity begins to rise faster than the shear thickening counterpart, but they have the same equilibrium height with the Newtonian liquid due to self-retardation (Digilov, 2008). For power-law fluids in conical tubes, asymmetric flow times have been found from different ends of the tubes, and the asymmetry is strengthened by shear-thinning fluid but weakened by the shear-thickening fluid (Berli and Urteaga, 2014). A comprehensive model is developed to predict the permeability of power-law fluid flow in fractal-like tree network, and the permeability is found to be a function of the branching diameter ratio, the branching length ratio, the total number of branching levels, and the power exponent of power-law fluids (Wang and Yu, 2011).

Control and acceleration of capillary flow is always advantageous for the applications mentioned above. The manipulated capillary flow in non-uniform porous structures, in terms of the evolution of flow distance to time, deviates from the classical Washburn equation based on uniform tubes (Reyssat et al., 2008, 2009; Shou et al., 2014b, 2014c). As well, the fastest capillary flow on the basis of Newtonian fluids has been found in the optimally designed tubes and porous systems (Shou and Fan, 2015; Shou et al., 2014b, 2014c, 2014d). However, to our best of knowledge, less studies concern the development and optimization of the fast capillary-driven power-law fluids. In this context, we aim for finding the minimum penetration time required for a given penetration distance, from a straight tube to a tree network. Both the straight tube and the tree network, which contain sub-tubes with different radii and lengths, are fundamental elements of various applications. More specifically, the straight tubes with varying local radii and lengths are often used in microfluidics devices (Stone et al., 2004) and found in general porous media (Yun et al., 2008), while the tree networks conduct blood in body as vessels (Murray, 1926) and drainage water/oil mixtures in forms of rock fractures (Wang and Yu, 2011). The investigation of transport in tree networks was dated back to 1926, when Murray (1926) found the optimal radius ratio between parent and daughter branches in a cardiovascular system restricted by energy expenditure. Based on the contructal theory (Reis, 2006), the minimization of flow resistance in a simple tree network under the given volume yields the same optimal radius ratio as Murray's law (Bejan et al., 2000), as the volume constraint is equivalent to the constraint of energy expenditure considered by Murray (1926). Even when the number of branch levels increases, the optimal radius ratio stays a constant (Kou et al., 2014). Moreover, the tree network requires a lower pumping power for transport (Chen and Cheng, 2002). In this work, we focus on investigating power-law fluids in the tree network, whereas the tree network becomes a straight tube when the branching number is unit.

2. Model generation

Capillary fluid, either Newtonian or non-Newtonian, moves spontaneously in tiny tubes due to the pressure of cohesion and adhesion. The capillary flow accelerates with the decrease in tube size as capillary force or capillary pressure increases. Conversely, the flow slows down with decreasing tube size, generating higher flow resistance based on Eq. (1). In a uniform tube, the time for a given flow distance is monotonously dependent on the tube size or the microstructure, in analogy to Washburn equation. In the tree network and the straight tube, the sub-tubes have different radii and lengths, whereas the capillary pressure and the viscous resistance have different sensitivity to microstructures of the local sub-tubes at different levels. Therefore, it is possible to find the minimum penetration flow time in the tree network as well as the straight tube with the dynamic competition between capillary and viscous effects. We investigate here the power-law flow through a multi-level Y-shaped tree network and a multi-section straight tube constrained by fixed volume and length. The gravitational force, the liquid evaporation, and the effect of meniscus shape is neglected, and a large aspect ratio of length to radius of the tubes is assumed. The fluids are in the continuum regime, as the tubes concerned in this work are much larger than the size of liquid molecules.

A Y-shaped tree network and a parallel tube net with the same volume and length are shown in Fig. 1. The parallel tube net is used as a control sample, as the flow time for a given total penetration distance (i.e., the tube length) is constant in a uniform tube. The influence of the branching angle α is eliminated, since all sub-tubes in the tree network have the same branching angle with the parallel tube net. In analogy to Ref. Chen et al. (2007), the number of parallel tubes is assumed to be equal to that of outlet tubes of the tree network, viz., m. In Fig. 1, m is equal to 2, while the tree network becomes a straight tube composed of two successive sub-tubes when $m=1$. The lower sub-tube at the parent branching level is named the first sub-tube, and the upper sub-tubes at the daughter branching level are named the second sub-tubes. We define the radius ratio and the length ratio between the second to the first sub-tubes

$$e=\frac{R_2}{R_1} \tag{2}$$

and

$$f=\frac{h_2}{h_1}, \tag{3}$$

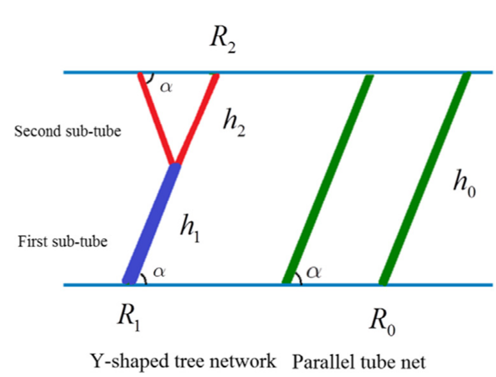

Fig. 1. Illustration of a Y-shaped tree network (left) and a parallel tube net (right).