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Numerical simulations of a coupled radiative–conductive heat transfer model using a modified Monte Carlo method

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ABSTRACT

Radiative–conductive heat transfer in a medium bounded by two reflecting and radiating plane surfaces is considered. This process is described by a nonlinear system of two differential equations: an equation of the radiative heat transfer and an equation of the conductive heat exchange. The problem is characterized by anisotropic scattering of the medium and by specularly and diffusely reflecting boundaries. For the computation of solutions of this problem, two approaches based on iterative techniques are considered. First, a recursive algorithm based on some modification of the Monte Carlo method is proposed. Second, the diffusion approximation of the radiative transfer equation is utilized. Numerical comparisons of the approaches proposed are given in the case of isotropic scattering.

1. Introduction

The study of the coupled heat transfer [1–3] where the radiative and conductive contributions are simultaneously taken into account is important for many engineering applications. So, Andre and Degiovanni [4,5], Banoczi and Kelley [6], and Klar and Siedow [7] have studied the thermal properties of some semi-transparent and insulating materials in the context of a coupled radiative–conductive model. The mathematical treatment of this nonlinear model is studied in [8–11]. In [8], Siewert and Thomas use the simple iteration method and a computationally stable version of the P_N approximation. In work [9], Siewert has applied the Newton iteration method instead of the simple iteration procedure. This allows the author to calculate some numerical examples which are not feasible using the simple iteration method (compare [8]). Kelley has provided existence and uniqueness theorems for the considered problem in the case of isotropic scattering and non-reflecting boundaries [10]. An analytical version of the discrete-ordinates method along with Hermite's cubic splines and Newton's method to solve a class of coupled nonlinear radiation–conduction heat transfer problems in a solid cylinder is proposed in [11]. The algorithm is implemented to establish high-quality results for various data sets which include some difficult cases.

In our paper, some iterative algorithm for solving this problem is considered. For the calculation of solutions of the radiative transfer equation, two ways are used. The first approach proposed by the authors utilizes a recursive algorithm based on some modification of the Monte Carlo method. This algorithm suits for the application of parallel calculations, and hence it can provide a good accuracy within a reasonable computing time. The second approach uses the diffusion approximation of the radiative transfer equation. It is shown that using this approximation gives a good description of the solution behavior. A numerical comparison of the approaches proposed is done in the case of isotropic scattering and reflecting boundaries. The calculations are implemented on a computer cluster of the Technical University of Munich using the technology of parallel computing supported by the application programming interface OpenMP.

2. Problem formulation

Let us consider the coupled radiative–conductive heat transfer problem which is formulated as in [8,9]. The equation of the radiation transfer for a homogenous layer is written in the normalized form as

$$\mu I_\tau(\tau,\mu) + I(\tau,\mu) = \frac{\omega}{2}\int_{-1}^{1} p(\mu,\mu')I(\tau,\mu')d\mu' + (1-\omega)\Theta^4(\tau), \quad (1)$$

where $I(\tau,\mu)$ is the normalized density of the radiation flux at the point $\tau \in [0,\tau_0]$ in the direction which angle cosine with the positive

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Nomenclature

A	an integral operator
B	operator of reflection
C_b	Banach space of bounded and continuous functions
D	a functional class
I	normalized density of the radiation flux
$I^{\langle j \rangle}$	radiation flux in the jth step of the iterative procedure
I_n	radiation flux in the nth step of the recursive procedure
h	input radiation flux
L	a linear operator
M	number of recursive trajectories
N	number of summands of the truncated Neumann series
N_c	conduction-to-radiation parameter
p	phase function
S	an integral operator
T	operator of the Neumann series
X	the set of optical and angular variables
Greek symbols	
α	iteration parameter
Γ^-	a set used in the definition of boundary conditions
Γ^+	a set used in the definition of boundary conditions
Θ_1	normalized temperature on the left boundary
Θ_2	normalized temperature on the right boundary
Θ	normalized temperature
$\Theta^{\langle j \rangle}$	temperature in the jth step of the iterative procedure
ε_1	emissivity coefficient of the left boundary
ε_2	emissivity coefficient of the right boundary
ω	albedo of single scattering
μ	angular variable
τ	optical depth (point of the layer)
τ_0	optical thickness of the layer
ξ	boundary point
ρ_1^d	coefficient of diffuse reflection of the left boundary
ρ_2^d	coefficient of diffuse reflection of the right boundary
ρ_1^s	coefficient of specular reflection of the left boundary
ρ_2^s	coefficient of specular reflection of the right boundary
ϕ_0	diffuse approximation of the average flux

direction of the axis τ is $\mu \in [-1,1]$; $\omega < 1$ the albedo of single scattering; $p(\mu,\mu')$ the phase function; $\Theta(\tau)$ the normalize temperature. Note that the case of non absorbing media ($\omega = 1$) is excluded from the consideration as unrealistic. Introduce the following sets for the definition of boundary conditions:

$$\Gamma^- = (\{0\} \times (0,1]) \cup (\{\tau_0\} \times [-1,0)),$$
$$\Gamma^+ = (\{0\} \times [-1,0)) \cup (\{\tau_0\} \times (0,1]).$$

We supply Eq. (1) with the boundary conditions

$$I(\xi,\mu) = h(\xi) + (BI)(\xi,\mu), \quad (\xi,\mu) \in \Gamma^-, \tag{2}$$

where the function h and the operator B are defined by

$$h(0) := \varepsilon_1 \Theta_1^4, \quad (Bf)(0,\mu) :$$
$$= \rho_1^s I(0,-\mu) + 2\rho_1^d \int_0^1 I(0,-\mu')\mu' d\mu', \quad \mu > 0,$$

$$h(\tau_0) := \varepsilon_2 \Theta_2^4, \quad (Bf)(\tau_0,\mu) :$$
$$= \rho_2^s I(\tau_0,-\mu) + 2\rho_2^d \int_0^1 I(\tau_0,\mu')\mu' d\mu', \quad \mu < 0.$$

Here, Θ_1 and Θ_2 are the normalized temperatures on the boundaries; ρ_i^s and ρ_i^d the coefficients of specular and diffuse reflections, respectively; $\varepsilon_i = 1 - \rho_i^s - \rho_i^d$ the emissivity coefficients for the boundary surfaces. It is assumed that $\varepsilon_1, \varepsilon_2 > 0$, which provides the estimate $\|B\| < 1$ (see Section 3). Note that the first summand on the right-hand side of the definition of the operator B describes the contribution of the specular reflection, whereas the second one describes the contribution of the diffuse reflection.

The equation of the conductive heat transfer is written as

$$\Theta''(\tau) = \frac{1}{2N_c}\left(\int_{-1}^1 I(\tau,\mu)\mu d\mu\right)', \tag{3}$$

and N_c is the conduction-to-radiation parameter [8]. For Eq. (3), we set the following boundary conditions:

$$\Theta(0) = \Theta_1, \quad \Theta(\tau_0) = \Theta_2. \tag{4}$$

For finding the solution of system (1)–(4), we will use a simple iteration method with parameter. According to that, choose an initial approximation of the temperature $\Theta(\tau)$ (for example, the linear approximation which corresponds to zero value of the right-hand side of (3)) and denote it as $\Theta^{\langle 0 \rangle}(\tau)$. Then, substitute $\Theta^{\langle 0 \rangle}(\tau)$ into (1) instead of the function $\Theta(\tau)$, find the solution of the problem (1) and (2), and denote it as $I^{\langle 1 \rangle}(\tau,\mu)$. Then, find the solution of the problem (3) and (4) under the given function $I^{\langle 1 \rangle}(\tau,\mu)$ and denote it as $\tilde{\Theta}^{\langle 1 \rangle}(\tau)$. Choose a small positive real α and set $\Theta^{\langle 1 \rangle}(\tau) = \alpha\tilde{\Theta}^{\langle 1 \rangle}(\tau) + (1-\alpha)\Theta^{\langle 0 \rangle}(\tau)$ to be the next approximation of $\Theta(\tau)$. Then, put $\Theta^{\langle 1 \rangle}(\tau)$ instead of the function $\Theta(\tau)$ into Eq. (1), find the next approximation $I^{\langle 2 \rangle}(\tau,\mu)$, and so on. Thus, in the jth step, we use the functions $\Theta^{\langle j-1 \rangle}(\tau)$ and $\tilde{\Theta}^{\langle j \rangle}(\tau)$ to determine the next approximation of the function $\Theta(\tau)$ by the following formula:

$$\Theta^{\langle j \rangle}(\tau) = \alpha\tilde{\Theta}^{\langle j \rangle}(\tau) + (1-\alpha)\Theta^{\langle j-1 \rangle}(\tau). \tag{5}$$

The main complexity in the numerical realization of this iterative method is finding the solution of the radiative transfer Eq. (1). For its treatment, we will mainly use a recursive algorithm based on the Monte Carlo method. As alternative, we will construct a diffusion approximation of Eq. (1) (P_1 approximation). We will compare the results of these approaches with the numerical data from [8,9].

3. Solvability of the radiative transfer equation

Let us consider the problem (1) and (2). We assume that the function $\Theta(\tau)$ is nonnegative, and $\Theta(\tau) \in C_b(0,\tau_0)$, where $C_b(X)$ is the Banach space of functions bounded and continuous on X with the norm $\|\varphi\|_{C_b(X)} = \sup_{x \in X} |\varphi(x)|$. Also, let $p(\mu,\mu') \in C_b(\Omega \times \Omega)$, where $\Omega = [-1,0) \cup (0,1]$, and

$$\frac{1}{2}\int_{-1}^1 p(\mu,\mu')d\mu' = 1.$$

Note that the operator $B : C_b(\Gamma^+) \to C_b(\Gamma^-)$ is linear, bounded, nonnegative, and

$$\|B\| \le \max_i(\rho_i^s + \rho_i^d) < 1.$$

Denote $X = (0,\tau_0) \times ([-1,0) \cup (0,1])$. We define a class $D(X)$ where solutions I of the problem (1) and (2) are sought.

A function $I(\tau,\mu)$ belongs to $D(X)$, if the following properties hold:

(1) $I(\tau,\mu)$ is absolutely continuous in $\tau \in (0,\tau_0]$ for all $\mu > 0$, and absolutely continuous in $\tau \in [0,\tau_0)$ for all $\mu < 0$;