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How to estimate added mass of a spherical cap body: Two approaches

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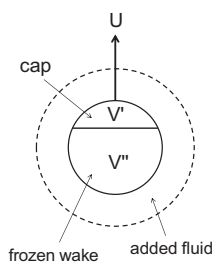
HIGHLIGHTS

- Dispersed flow modelling.
- Estimating added mass of a cap-body.
- Analogy between two different models.

GRAPHICAL ABSTRACT



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ABSTRACT

An analogy is established between two different approaches for estimating the added mass coefficient of a spherical cap body. These two originate from very different assumptions but yield very similar results. Here we try to explain why it is possible.

1. Introduction

Multiphase flow systems are widespread in occurrence within possibly all branches of chemical technology where inter-phase contact and reaction are desired. Deformable fluid particles (bubbles, drops) dispersed in carrying media acquire variety of forms, one of which is the spherical cap geometry. In the classic 'shape map' (diagram 2.5 in Clift et al., 1978), the cap is located in the top-right region, which can be considered as a limit of large Eotwos and Reynolds numbers and low Morton number. These shapes are well-known and have been studied intensely, both by engineers and fluid mechanists, to understand their basic features: shape properties, rise velocity, flow field around, typical flow regimes, wake dynamics, mass and heat transport (see e.g. the early review by Wegener and Parlange, 1973). The basic scaling for the terminal speed $U \sim (g.a)^{0.5}$ was known already in more than 6 decades ago and was many times approved, discussed, and extended (e.g. Davenport et al., 1967; Guthrie and Bradshaw, 1973; Joseph, 2003). Considerable attention was paid to the wakes behind cap bodies, their structure and properties, because the wakes cause mixing and contribute to transport phenomena as carriers of the surrounding medium (e.g. Wegener et al., 1971; Coppus et al., 1977; Komasawa et al., 1980; Fan and Tsuchiya, 1990). One of the most useful and practical outputs is the information about the mass and heat transfer, which can then be used for design purposes (e.g. Brignell, 1974; Weber, 1975; Coppus and Rietema, 1981; Kendoush, 1994). The cap bubbles are very often studied in fluidized beds, where they abound in the so-called 'bubbling regime', which was the subject of many studies (e.g. Jackson, 2000 and other earlier standard texts on fluidized bed),

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and still is a vital part of current modelling efforts (e.g. Hoogstraten et al., 1998; Villa Briongos et al., 2011). On the more theoretical, fluid mechanical side, analytical studies were performed when possible, typically in the laminar case (e.g. Parlange, 1969, Dorrepaal et al. 1976) or in the potential case (e.g. Davies and Taylor, 1950), possibly including the viscosity (Joseph, 2003). The cap shape may become unstable and can change into another geometry, for instance the skirt can be formed (e.g. Hnat and Buckmaster, 1976) or the lenticular shape with toroidal wake of satellite bubbles (Landel et al., 2008), or the bubble itself can become an annular object (Bonometti and Magnaudet, 2006). With the help of the direct numerical simulations, the ranges of the intermediate Reynolds number can be approached and maps of different types of solutions and their stability can be investigated (e.g. Feng 2007).

Regardless the point of view, be it motivated theoretically or practically, one phenomenon related to dispersed particles is important on both accounts: the added mass effect. The inertial forces related to added mass are induced at unsteady motions of bubbles and drops. The added mass coefficients enter the formulas for the kinetic energy of the dispersed systems. The added mass phenomena are related to the Darwin drift phenomenon where parts of the dispersion are physically moved from one place to another, resulting in tangible mixing of the medium. Despite its relevance to the behaviour of cap bodies submersed in fluids, the added mass has not been paid the adequate attention so far. To our knowledge, the singular contribution is that of (Kendoush (2003 – denoted as KEN), where an approximate analytical method was developed for estimating the added mass coefficient C of a spherical-cap shape. Because of the obvious lack of a simple and practical formula, suitable for rough estimates for engineering purposes, for some time we have been using our common-sense guess-relation based on a naive concept of a 'frozen wake'. In our recent contribution (Simcik et al., 2014 – denoted as SPR) we pointed out that despite their obvious differentness and disparity, these two approaches, SPR and KEN, give very similar estimates (within few %). This fact was left unattended, as a mere curiosity.

The goal of this short contribution is to offer an explanation, why the two different approaches yield so similar results.

2. Two methods of C estimation

The added mass coefficient C of a spherical-cap body in a uniform unbounded fluid is in the focus. C is generally defined as the ratio of the added fluid volume V_a to the body volume V_{body}. The added volume V_a relates to added mass m_a by the fluid density ρ, $m_a=\rho V_a$. The added mass relates to the flow energy E generated by the body accelerated from rest to velocity U by $E=(1/2)m_aU^2$ (e.g. Lamb, 1932, Batchelor, 1967).

2.1. Approach by SPR

This approach stems from a need to have a simple engineering formula for C, which meets few basic requirements: C is a monotonously increasing function of cap body flatness, C is 0.5 for the full sphere, C diverges for the flat disc, C is expressible in simple terms of the cap body geometry. This approach (Simcik et al., 2014) is based on a crude physical model whose definition sketch is drawn in Fig. 1a. The added fluid volume V_a is the sum of two contributions: that of a spherical body $(1/2)V$ and the volume of the 'frozen wake' V'' behind the cap body of volume V'. Simple relations hold: $V=V'+V''$ and $V=(4/3)\pi a^3$. With help of the cap-to-sphere volume fraction $f=V'/V$, the resulting added volume can be written as:

$$V_a=(1/2)V+(1-f)V \quad [\text{model SPR}] \tag{2.1.1}$$

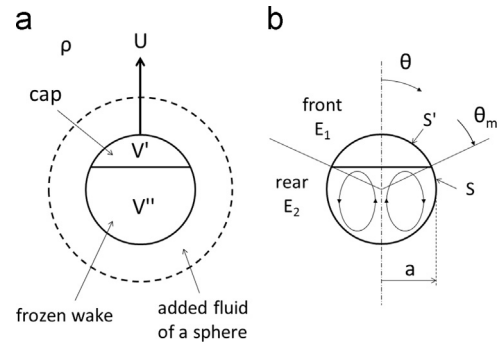

Fig. 1. Definition sketch for a spherical-cap body of velocity U moving through an infinite fluid. The spherical envelop has radius a, surface S, volume V. The cap body has spherical surface S', volume V' and its wake has volume V''. The surface ratio is $p=S'/S$ and the volume ratio is $f=V'/V$. The cap shape is determined by the subtending cap angle $\theta_m \in <0, \pi>$. The added mass m_a is depicted as the added volume of fluid V_a stuck to the body. (a) Model SPR. The added fluid volume V_a has two parts: that of the whole sphere (0.5 V) plus the 'frozen wake' behind the cap ($(1-f)V$). (b) Model KEN. The front region ahead of the cap for $\theta \in <0, \theta_m>$ has the single-sphere potential flow and energy E_1. The rear wake region for $\theta \in <\theta_m, \pi>$ has Hill's spherical vortex flow and energy E_2. The added mass is calculated from the flow energy, integrating the velocity field.

Dividing by the cap volume V' we obtain the coefficient in the form

$$C=(3/2-f)/f, \tag{2.1.2}$$

and with help of the full-sphere value C_0 it finally reads:

$$C=C_0(3-2f)/f. \tag{2.1.3}$$

2.2. Approach by KEN

The other approach is based on an approximate analytical model for the kinetic energy of the flow field due to the cap body (Kendoush, 2003). The flow is divided into two regions, demarcated by the cap angle θ_m, see the definition sketch in Fig. 1b.

In the *front* region, ahead of the body (surface S'), there is the potential flow corresponding to a single sphere. This region is defined by the cap angle $\theta \in <0, \theta_m>$ and its energy contribution E_1 is obtained by integrating the velocity field (see Eq. 5 in Kendoush (2003)),

$$E_1=(\pi/3)a^3\rho U^2k. \tag{2.2.1}$$

The geometrical weight factor $k \in <0, 1>$ is an increasing function of the cap angle. Denoting $c \equiv \cos\theta_m$, the formula for k reads:

$$k=(1/4)(2-c-c^3). \tag{2.2.2}$$

In the limit of $\theta_m=\pi$ (full sphere, $c=-1$), it gives the energy for an isolated sphere, which corresponds to the added volume $V_a=(1/2)V$. In the limit of $\theta_m=0$ (no cap, $c=1$) we have no energy as the body shrinks to zero.

In the *rear* region (volume V''), the wake behind the cap with $\theta \in <\theta_m, \pi>$, the presence of Hill's spherical vortex is assumed. The energy contribution E_2 carried by the vortex is calculated by integration (see Eq. 10 in Kendoush (2003)), for a range of the cap shapes $\theta_m \in <0, \pi>$, to result in the following expression:

$$E_2=(9\pi/14)a^3\rho U^2[1+c-(1/4)(1+c^3)+(1/8)(1-c^8)-(3/40)(1-c^{10})]. \tag{2.2.3}$$

In the limit of $\theta_m=\pi$ (no vortex, $c=-1$), it correctly gives $E_2=0$. In the limit of $\theta_m=0$ (full vortex, $c=1$) we have the full spherical vortex with energy $E_{Hill}=(27/28)\pi a^3\rho U^2$, which corresponds to the added volume $V_a=(81/56)V \approx 1.446$ V. The added mass coefficient $C=C(\theta_m)$ is then calculated with help of the flow energy (E_1+E_2) to obtain the