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The onset of natural convection in a horizontal fluid layer heated isothermally from below

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ABSTRACT

The onset of buoyancy-driven convection in an initially quiescent fluid layer confined between two horizontal plates is analyzed theoretically. In case of isothermal heating it is well known that convective motion sets in when the Rayleigh number Ra exceeds 1708. For $Ra > 1708$, there are three characteristic times t_c, t_D and t_u which represent respectively, the critical time to mark the onset of intrinsic instability, the detection time of motion, and the undershoot time in a plot of the heating rate versus time. These characteristic times are analyzed by employing the numerical method under the single mode of instabilities and fitting some experimental t_u-values. The new measures to represent t_c and t_D are suggested, based on the growth rates of fluctuations. It is interesting that t_c is the invariant but the predicted t_D- and t_u-values are dependent upon the magnitude of initial conditions forced. It is shown that for the isothermally heated system of a large Prandtl number the relation of $t_u \cong 7t_c$ agrees well with the available experimental t_u-values for $Ra > 10^5$ and t_D is located between t_c and t_u. This paper removes the confusion among the characteristic times, t_c, t_D and t_u in the literature on stability. Also the boundary-layer instability model is discussed in order to analyze turbulent thermal convection heat transfer characteristics in the fully developed state, based on the present numerical predictions.

1. Introduction

Buoyancy-driven convection abounds in nature. Its role in heat exchangers and also in processes such as chemical vapor deposition, crystal growth and electroplating is well known. Therefore, it is very important to predict transfer properties in a number of processes.

Let us consider an initially quiescent, isothermal fluid layer placed between two infinite horizontal plates. The bottom plate is heated isothermally, starting from time $t = 0$, whereas the top one is kept at the initial temperature. With very slow heating, the conduction temperature profile becomes linear as time passes and steady state buoyancy-driven convection sets in at the critical Rayleigh number $Ra_c = 1708$. On the other hand, with high heating rates ($Ra \gg Ra_c$) the temperature field becomes nonlinear and time-dependent. Even though convection is detected experimentally, there is no convincing evidence on the origin of the observed motion. During the heating period some thermal noise encountered in the experimental environment may lead to thermal convection. But we do not know what it is and also when it is initiated. Therefore, for $Ra \gg Ra_c$ theoretical pursuit toward nonlinear analysis on the temporal evolution of thermal convection is formidable. In this connection many researchers have proposed various models. Morton [1] first suggested the quasi-static model, where the basic temperature field is frozen at each instant. Subsequent models were introduced by Choi et al. [2,3] and Kim and Choi [4]. But these are not definitive and moreover available experimental data are limited to those of the undershoot time t_u in systems of large Prandtl number Pr [5,6]. Later, for the forced convection systems heated from below with constant heat flux, Park et al. [7,8] analyzed the onset and growth of the fluctuation by solving nonlinear, single mode disturbance equation numerically under the proper initial condition.

In this study, Park et al.'s [7,8] approach is extended into the initially quiescent fluid layer heated isothermally from below. The nonlinear equations of motion and energy are solved numerically by employing the finite volume method (FVM), and the growth of fluctuations is traced with time. Based on the numerical results, the characteristic times (t_c, t_D, t_u) and also the fully-developed turbulent heat transfer rate will be discussed and compared with available experimental data.

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Nomenclature

a	dimensionless wave number
A	amplitude of temperature fluctuation
B	amplitude of velocity fluctuation
g	gravitational acceleration [m/s^2]
H	height of layer [m]
k	thermal conductivity [N/s K]
Nu	Nusselt number
P	pressure [N/m^2]
Pr	Prandtl number, ν/α
$r_{0,T}$	growth rate of mean temperature
$r_{1,T}$	growth rate of temperature fluctuation
$r_{1,V}$	growth rate of velocity fluctuation
q	heat flux [N/m s]
Ra	Rayleigh number, $g\beta\Delta TH^3/(\alpha\nu)$
t	time [s]
T	temperature [K]
$\mathbf{u}$	dimensionless velocity vector, $\mathbf{U}H/\alpha$.
$\mathbf{U}$	dimensional velocity vector [m/s]

Greek symbols

α	thermal diffusivity [m^2/s]
β	thermal expansion coefficient, 1/K
ν	kinematic viscosity [m^2/s]
Δ	conduction-layer thickness [m]
θ	dimensionless temperature, $(T-T_i)/(T_w-T_i)$
ρ	density [kg/m^3]
τ	dimensionless time, $t\alpha/H^2$

Subscripts

c	critical state
i	initial state
rms	root-mean-square quantity
w	bottom wall
0	basic state
1	perturbed state
∞	fully-developed state

2. Governing equations

2.1. Boussinesq approximation

The system considered here is a Newtonian fluid layer with uniform initial temperature of T_i (for time $t < 0$). For $t \geqslant 0$, the bottom plate of horizontal fluid layer of thickness H is suddenly heated to T_w, whereas the upper boundary is kept at the initial one T_i. For high $\Delta T = T_w - T_i$, buoyancy-driven convection will set in at a certain time. The dimensionless governing equations for the flow and temperature fields are expressed using the Boussinesq approximation by

$$\nabla \cdot \mathbf{U} = 0, \tag{1}$$

$$\left\{\frac{\partial}{\partial\tau} + \mathbf{U}\cdot\nabla\right\}\mathbf{U} = -\nabla P + Pr\nabla^2\mathbf{U} + \mathrm{Pr}Ra\theta\,\mathbf{e}_z, \tag{2}$$

$$\left\{\frac{\partial}{\partial\tau} + \mathbf{U}\cdot\nabla\right\}\theta = \nabla^2\theta, \tag{3}$$

where $\mathbf{U}$, P, θ, and $\mathbf{e}_z$ represent the velocity vector, dynamic pressure, temperature, and unit vector in the dimensionless vertical z-direction, respectively. The time, distance, velocity and pressure have, respectively, the scales of H^2/α, H, α/H and $\rho\alpha^2/H^2$. The temperature has been nondimensionalized as $\theta = (T - T_i)/\Delta T$, and ρ denotes the fluid density. The important parameters to describe the present system are the Rayleigh number $Ra = g\beta\Delta TH^3/(\alpha\nu)$, the Prandtl number $Pr = \nu/\alpha$, and the Nusselt number $Nu = q_wH/(k\Delta T)$, where g, β, α, ν, q_w and k denote the gravitational acceleration constant, thermal expansivity, thermal diffusivity, kinematic viscosity, bottom heat flux, and thermal conductivity, respectively. For conduction the time-dependent temperature profile is well known (e.g., see Foster [9]).

2.2. Mean-field equations and temporal growth rates

For $Ra \geqslant 1708$ the temperature and velocity fields are decomposed into the horizontal mean and its fluctuations:

$$\theta = \langle\theta\rangle + \theta', \quad \mathbf{U} = \langle\mathbf{U}\rangle + \mathbf{U}'. \tag{4a, b}$$

The fluctuations depend on τ, x, y and z, where x and y are the Cartesian coordinates on the horizontal plane. With infinitesimal fluctuations the conduction state is dominant and linear theory is applicable.

In the present system, thermal convection sets in due to buoyancy forces and its dimensional magnitude F_B is represented by

$$F_B = \rho g\beta|T - T_i|, \quad F_B = F_{B,0} + F_{B,1}, \tag{5a, b}$$

which are produced by temperature variations. The buoyancy forces based on the mean temperature and its fluctuations can be written as $(F_{B,0}, F_{B,1}) = (\langle\theta\rangle, |\theta'|)\rho g\beta\Delta T$. These buoyant forces are closely related with the so-called thermal energy. To examine the temporal behaviors of thermal instabilities, the following temporal growth rate of the mean value and that of its fluctuations are defined, respectively:

$$r_{0,T} = \frac{1}{\langle\theta\rangle_{rms}}\frac{d\langle\theta\rangle_{rms}}{d\tau}, \quad r_{1,T} = \frac{1}{\theta'_{rms}}\frac{d\theta'_{rms}}{d\tau}. \tag{6a, b}$$

Here the subscript '*rms*' refers to the root-mean-square quantity, i.e., $(g)_{rms} = [\int_V |(g)|^2 dV/V]^{1/2}$, where V represents the volume of the system considered. The maximum temperature or thermal energy may be used instead of the *rms* values of temperature fields. Similarly, the temporal growth rate of velocity fluctuations is defined as

$$r_{1,V} = \frac{1}{|\mathbf{U}'|_{rms}}\frac{d|\mathbf{U}'|_{rms}}{d\tau},$$
$$|\mathbf{U}'|_{rms} = \left[\frac{1}{V}\int_V (u'^2 + v'^2 + w'^2), dV\right]^{1/2}. \tag{7a, b}$$

With $r_{0,T} \gg r_{1,T}$, temperature fluctuations are expected to be several orders of magnitude smaller than that of the mean temperature. Here it is assumed that the system is unstable only when temperature fluctuations grow faster than the mean temperature. With $r_{1,T} > r_{0,T}$, temperature fluctuations will grow to a detectable magnitude. Therefore, for a given Ra we employ the stability criterion suggested first by Choi et al. [3]:

$$r_{1,T} = r_{0,T} \quad \text{with} \quad r_{1,V} \geqslant 0 \quad \text{at} \quad \tau = \tau_c, \tag{8}$$

which marks the onset condition of intrinsic instability at the earliest time τ_c with the critical dimensionless wavenumber a_c. A positive growth rate of each quantity ($r_{1,T}, r_{1,V} > 0$) is tolerated as long as temperature fluctuations are not growing faster than the basic ones ($r_{1,T} \leqslant r_{0,T}$). If any flow initiated is simply an induced one without