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Finite element investigation of the flow and stress patterns in conical hopper during discharge

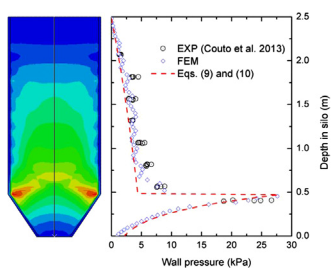
Q.J. Zheng [a,b], A.B. Yu [a,*]

[a] Department of Chemical Engineering, Monash University, Clayton, Vic. 3800, Australia
[b] Laboratory for Simulation and Modelling of Particulate Systems, School of Materials Science and Engineering, The University of New South Wales, Sydney, NSW 2052, Australia

HIGHLIGHTS

- An Eulerian-formulation finite element method is used to describe granular flow in hopper.
- This FEM outperforms the ordinary Lagrangian computation in resolving the mesh distortion problem.
- Satisfactory results are obtained in mass and funnel flow modes, mass flowrate and wall pressure.

GRAPHICAL ABSTRACT



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ABSTRACT

Precise evaluation of the dynamics of granular material during hopper discharge, particularly the velocity field coupled with the stress field, has been an important area of research for many years. In this paper, a finite element method (FEM) based on the Eulerian formulation is described and validated to meet this need. It is demonstrated that this method outperforms the ordinary Lagrangian-formulation method in resolving the problem of mesh distortion, thereby is capable of simulating the complete emptying process of a hopper. On this basis, various discharge behaviours of a conical hopper are studied, including the mass and funnel flow modes, the mass flow rate and the wall pressure. The results are in general agreement with those from experiments and recognized correlations in all the examined aspects, which validates the applicability of the Eulerian FEM. This continuum method ought to be of practical significance because it is computationally tractable, and viable in dealing with complex silo geometries and variable flow patterns of granular materials.

1. Introduction

Flow of granular materials in hoppers is a common occurrence in many industrial applications. This confined flow has some unique flow and stress characteristics, which should be fully understood in order to design a device satisfying operational and safety requirements. It has been a subject of research over the past century as seen from reviews (Nedderman et al., 1982; Tuzun et al., 1982; Drescher, 1991; Nedderman, 1992; Roberts, 1998; Rotter, 2001).

A few salient representative studies had been done by Janssen (1895), Beverloo et al. (1961) and Jenike (1961, 1964), which have shed light on the general flow behaviours. However, how to precisely evaluate the velocity and stress fields within a hopper continues to present a challenge in practice. One reason for it is that the classic theories have considered solely concentric

* Corresponding author. Tel.: +61 3 99050582; fax: +61 3 99055686.
E-mail address: Aibing.Yu@monash.edu (A.B. Yu).

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geometries such as conical and wedge-shaped hoppers, where the centerlines of the outlet and hopper coincide. But owing to some particular needs, complex or eccentric hoppers with an off-centered outlet are also being widely used in industrial applications (Ketterhagen and Hancock, 2010), which may have more complicated flow and load behaviours (Ramírez et al., 2010). Furthermore, the previous hopper theories were all derived under certain assumptions, which may or may not hold depending on the situation. For example, Ding et al. (2013) pointed out that the pressure theories (Janssen, 1895; Walker, 1966; Walters, 1973) are mainly applicable to steep hoppers because the effect of wall friction in such hoppers is fully developed during the discharge as assumed in the theory, whereas for shallow hoppers with only partially mobilized wall friction, considerable emendations to the theories are required.

In general, two types of numerical methods are used in the modelling and simulating granular flow: discrete and continuum. In continuum-typed methods, granular material is treated as a continuum medium assuming that the system scale is always much greater than component particles or grains. Its dynamics is governed by conservation equations of mass, momentum, and energy, with a constitutive model describing its intrinsic characteristics as well as the initial and boundary conditions. The granular flow is thus modelled at a macroscopic or global scale. Discrete methods, in particular the so-called discrete element method (DEM), can on the other hand achieve a particle-scale simulation of particulate system. DEM simulates the motion of each individual particle in a system based on the second Newtonian law and some practical contact or non-contact force models (Zhu et al., 2007, 2008). Unlike common theoretical treatments, DEM does not enforce assumptions on constitutive relationships or hopper geometry and can provide important microdynamic information such as the trajectories of and transient forces acting on individual particle. Consequently, it has been increasingly employed in recent decades to explore hopper behaviours, e.g. the mass discharge rate (Zhu and Yu, 2004; Anand et al., 2008), particle flow patterns (Ketterhagen et al., 2009) and internal bulk stresses (Langston et al., 1995; Rotter et al., 1998; Zhu and Yu, 2002). Nevertheless, it is known that to a large degree, DEM simulation can only deal with small-scale systems because it is computationally very demanding. How to overcome this problem remains a challenge, although some measures such as the multi-domain continuum and discrete methods (Parisi et al., 2004; Rousseau et al., 2009; Nitka et al., 2011) are proposed to mitigate this computational tension.

As a continuum approach, Finite Element Method (FEM) also prevailed in previous hopper investigations. FEM associated with elastoplastic constitutive theories can give satisfactory predictions of the internal stress and wall pressure in hoppers (Ooi and Rotter, 1990; Rotter et al., 1998; Goodey et al., 2003, 2006; Goodey and Brown, 2004; Vidal et al., 2006, 2008; Ding et al., 2013). However, its application to reproduce the flow conditions of granular material is usually not easy (Rotter et al., 1998; Sanad et al., 2001; Tejchman and Klisiński, 2001) and requires some particular schemes or treatments such as the re-meshing and re-zoning scheme (Sanad et al., 2001), adaptive meshing technique (Yang et al., 2011), viscoplastic granular fluid models (Haussler and Eibl, 1984; Karlsson et al., 1998; Elaskar et al., 2000; Böhrnsen et al., 2004), smoothed particle hydrodynamics (SPH) method (Sugino and Yuu, 2002), and material point method (MPM) (Wieckowski et al., 1999).

In our recent study (Zheng and Yu, 2014), an Eulerian-formulation FEM was implemented in the analysis of a representative problem, i.e. stress dip beneath sandpile. This approach can describe the dual solid- and fluid-like behaviours of granular material and satisfactorily reproduce the critical stress dip phenomenon under various conditions. It not only explains the mechanism of dip but may also open up a new direction to describe granular materials in nature and many industrial processes. The goal of this paper is to extend its application to a common industrial process, i.e. hopper discharge; and to compare the FEM results of granular dynamics with the state-of-art hopper theories and experimental measurements. The paper is organized as follows. First, an elaborate description of the computational methods as well as the constitutive relationship and boundary conditions used in simulations are given in Section 2. The simulation results of mass flow rate, flow pattern and wall stress are presented and discussed in Section 3. Finally, the main conclusions of this work are given in Section 4.

2. Set up for numerical simulation

2.1. Mohr–Coulomb elastoplastic relationship

It is known that the robustness of a continuum approach depends largely on the choice of the constitutive model used for describing material behaviours. To date, a number of such constitutive models have been proposed for granular materials in the literature, ranging from the classic plastic theories (Coulomb, 1776; Drucker and Prager, 1952; Lade, 1977) to more recent polar elastoplastic theory (Tejchman and Wu, 1993), hypoplasticity theory (Wu et al., 1996), double shearing theory (Spencer and Hill, 2001) and others. But at the bulk scale where micromechanics such as shear banding is not concerned, the classic elastoplastic models are believed to be adequate for capturing stress characteristics of granular materials (Goodey et al., 2003, 2006; Zheng and Yu, 2014). Hence, this paper utilizes the traditional Mohr–Coulomb elastoplastic model to characterize the mechanical behaviours of particulate materials stored in a hopper. The model mainly consists of three parts: a linear isotropic elastic law to define the material behaviour at small loads, a yield criterion to determine the transition of material into yielding, and a non-associated plastic flow potential to determine the flow directions after material yielding. Exact expression of this Mohr–Coulomb elastoplastic model can be found in the literature (Abaqus 6.10, 2010) and is briefly described below.

This elastic behaviour is completely determined by Young's modulus E and Poisson's ratio ν in isotropic elastic law:

$$\sigma_{ij} = D^{el}_{ijkl}\varepsilon^{el}_{kl} \tag{1}$$

where σ_{ij} is the total stress; ε^{el}_{kl} is the elastic strain; and D^{el}_{ijkl} is the fourth-order tensor of elasticity.

The yield condition is given by

$$R_{mc}q - p\ \tan\ \phi - c = 0 \tag{2}$$

where

$$R_{mc} = \frac{1}{\sqrt{3}\ \cos\ \varphi}\ \sin\left(\Theta + \frac{\pi}{3}\right) + \frac{1}{3}\ \cos\left(\Theta + \frac{\pi}{3}\right)\tan\ \varphi \tag{3}$$

$p = -\frac{1}{3}trace(\sigma_{ij})$ is the first invariant of stress representing the equivalent pressure; $q = \sqrt{\frac{3}{2}(S_{ij}S_{ij})}$ is the Mises equivalent stress and S_{ij} is deviatoric stress; and φ and c are known as the angle of internal friction and the cohesion of granular material. Θ is the deviatoric polar angle and is defined by $\cos(3\Theta) = (r/q)^3$ where r is an invariant measure of deviatoric stress $r = \left(\frac{9}{2}(S_{ij}S_{jk}S_{ki})\right)^{1/3}$.

The flow potential G is chosen to be a hyperbolic function in the meridional stress plane and a smooth elliptic function in the deviatoric stress plane, defined mathematically by the following formula:

$$G = \sqrt{(\epsilon c|_0\ \tan\ \psi)^2 + (R_{mw}q)^2} - p\ \tan\ \psi \tag{4}$$