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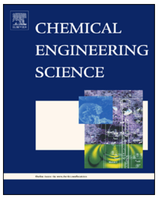

Finite element continuum modeling of vibrationally-fluidized granular flows

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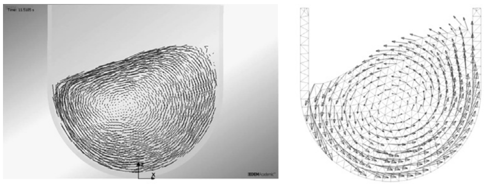
HIGHLIGHTS

- A continuum model was developed for a vibrationally-fluidized granular flow.
- The constitutive equations governing the continuum model were calibrated using DEM.
- The bulk flow velocities predicted by Lagrangian FE and DEM were in good agreement.
- The local impact velocity of the continuum model was inferred from the shear rate.

GRAPHICAL ABSTRACT

Discrete element model and Finite element continuum model.



Discrete element model

Finite element continuum model

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ABSTRACT

Discrete element modeling of granular flows is effective, but requires large numerical models. In an effort to reduce computational effort, this paper presents a finite element (FE) continuum model of the vibrationally fluidized granular flow produced by a tub vibratory finisher. The constitutive equations governing the continuum model were calibrated using the discrete element method (DEM). A constant stiffness was used for the normal contact of the polyurethane tub wall and the spherical steel media, and an effective coefficient of Coulomb friction was obtained by studying the variation of the shear and normal forces at different points along the tub wall using DEM simulations. The DEM predictions of the ratio of the tangential to normal contact forces showed that rolling of the media on the tub wall was much more common than sliding, and the average effective coefficient of friction was relatively uniform over the wall. Average values of the equivalent elastic and plastic properties of the media were used in the continuum model, neglecting the small local variations that existed within the vibrationally-fluidized bed. The bulk flow behavior of the equivalent continuum media was then studied using both Lagrangian and Eulerian FE formulations. The bulk flow velocities predicted by the Lagrangian approach were in good agreement with those obtained using DEM simulations over a wide range of tub wall amplitudes. The qualitative trend of the local impact velocity distribution at various locations in the tub predicted by the DEM agreed with the trend predicted by the continuum model using the shear rate as a measure of the granular temperature and hence local velocity. The FE simulations required approximately 8 times less computing time than the equivalent DEM.

1. Introduction

The flow of granular materials has been studied in non-fluidized beds such as in hopper discharge (Ketterhagen et al., 2007, 2009), rotating drums, conveyers, chutes and mixers (Yang et al., 2003; Jasti and Higgs, 2008), and in fluidized beds such as in vibratory finishers and vibrating sieves (Deen et al., 2007; An et al., 2009; Naeini and Spelt, 2009, 2011). Vibratory finishing is widely used to polish, burnish, harden, and clean metal, ceramic and plastic parts. In a tub vibratory finisher, the two-dimensional vibration of the walls produces a vibrationally-fluidized circulatory

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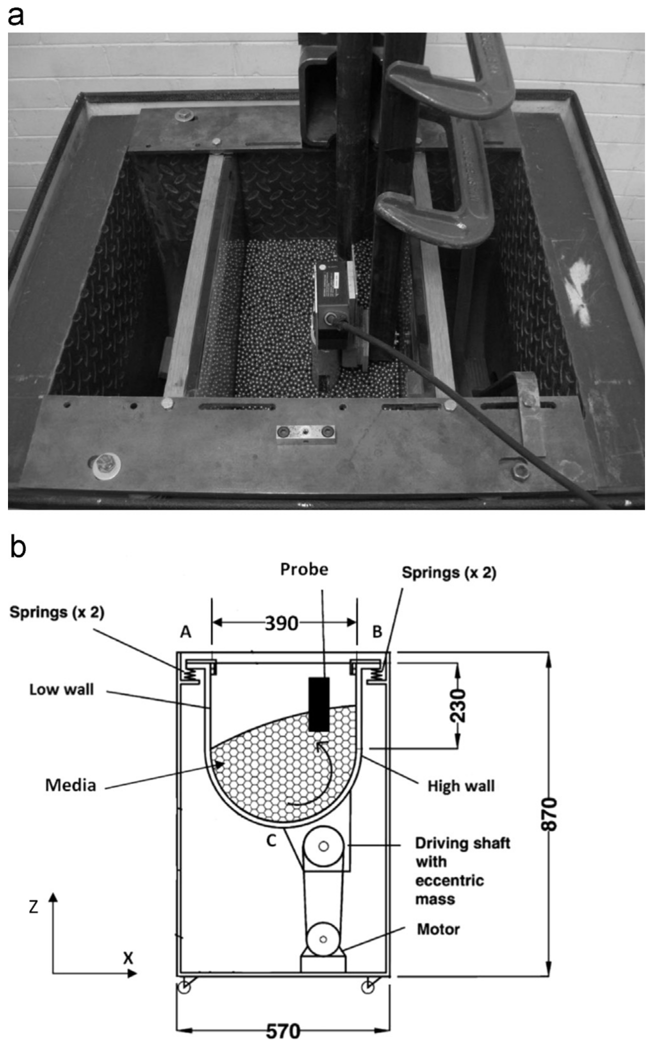

Fig. 1. (a) Photograph of the tub vibratory finisher including the laser probe used to measure the media impact velocities in Ref. Hashemnia et al. (2013), and two glass partitions within the tub. (b) Schematic of the tub vibratory finisher showing the bulk flow circulation direction from the side view. Dimensions in mm Hashemnia and Spelt (2014).

bulk flow of the media (Fig. 1). The granular media becomes fluidized as the vertical acceleration amplitude exceeds that of gravity, resulting in a marked decrease in the contact pressure (Naeini and Spelt, 2011). The media have both a large-scale bulk flow velocity resulting from the resultant shear force with the walls, and a local impact velocity during each vibration cycle (Hashemnia et al., 2013).

The erosive wear and plastic deformation of a workpiece within a vibratory finisher are determined mainly by the velocity, frequency, and direction of the impacts of the granular finishing media (Ciampini et al., 2007; Wang et al., 2000; Yabuki et al., 2002). Moreover, these impacts can lead to breakage of the finishing media. In processes such as drug tablet coating within rotating drums, and bulk materials handling, the impact of the granular products can contribute to their fragmentation (EDEM Website, 2012).

Generally, discrete element modeling (DEM) simulations (Munjiza, 2004; Zhu et al., 2008) give reasonable predictions of bulk flow velocity and volume fraction in both fluidized and non-fluidized granular flows (Tai et al., 2010; Majid and Walzel, 2009; Stewart et al., 2001). The local behavior of the non-fluidized granular flows has also been investigated experimentally and numerically (Ketterhagen et al., 2005, 2009; Freireich et al., 2009; McCarthy et al., 2010), while some studies have examined vibrationally-fluidized flows (Hashemnia et al., 2013; Ciampini et al., 2007; Wang et al., 2000; Yabuki et al., 2002; Hashemnia and Spelt, 2014; Xiang et al., 2010). Recently, it has been shown that DEM predictions of both bulk flow and local impact velocities in a tub vibratory finisher agreed well with measurements made using a high-speed laser displacement probe immersed in the flow (Hashemnia and Spelt, 2014).

Previous efforts to model granular flows using a continuum approach have focused on the bulk flow behavior of the non-fluidized granular media such as in the flow down inclined chutes (Kamrin, 2010; Jop et al., 2006; Andrade et al., 2012), plane shear flow (Midi, 2004; Cruz et al., 2005), flow in rotating drums (Forterre and Pouliquen, 2008; Midi, 2004), flow in annular shear cells (Kamrin, 2010; Forterre and Pouliquen, 2008; Midi, 2004), and silo discharge flows (Rycroft et al., 2009; Forterre and Pouliquen, 2008; Midi, 2004). In these cases of non-fluidized flows, constitutive equations were defined to describe the bulk flow of the granular materials under quasi-static and moderate flow (liquid-like) regions (Kamrin, 2010; Jop et al., 2006; Andrade et al., 2012). To the authors' knowledge, no papers have been published on continuum modeling of vibrationally-fluidized granular flows. The continuum models that have been used for quasi-static and moderate flow include different elasto-plastic formulations of the equivalent continuum media (Kamrin, 2010; Andrade et al., 2012; Andrade and Tu, 2009), and visco-plastic formulations considering only the plastic behavior of the equivalent media under time-varying shear deformation (Jop et al., 2006; Forterre and Pouliquen, 2008). In many cases, DEM was used to obtain the equivalent properties needed to model the granular flows as a continuum (Rycroft et al., 2009; Forterre and Pouliquen, 2008; Midi, 2004; Cruz et al., 2005; Andrade et al., 2011; Guo et al., 2012). For example, the equivalent stress tensor, pressure and shear rate at different points of a flowing granular bed were obtained using DEM, and were then used to determine the equivalent continuum media elastic and plastic properties (Rycroft et al., 2009; Andrade et al., 2011, 2012). Generally, the average streamlines, and hence the bulk flow behavior, determined through these continuum simulations have been in fairly good agreement with the predictions of discrete element modeling (Kamrin, 2010; Forterre and Pouliquen, 2008; Andrade et al., 2012). For example, Kamrin (2010) proposed an elasto-plastic constitutive law for use in a Lagrangian finite element model of granular flows in inclined chutes, rectangular silos and annular Couette cells. The predicted flow fields were compared with those calculated using DEM. Andrade et al. Andrade et al. (2011, 2012) used the same material law to model the static 3D compression of the sand particles. Forterre and Pouliquen (2008) used a visco-plastic constitutive law with the fixed-grid finite difference method, which is equivalent to an Eulerian mesh to simulate granular flows in the geometries of Kamrin (2010) and in rotating drums. They compared their results with experimental measurements and DEM predictions of bulk flow (Forterre and Pouliquen, 2008). The Lagrangian formulation has been used in most of the papers that modeled granular flows using the finite element method (Kamrin, 2010; Andrade and Tu, 2009).

The present work modeled the tub vibratory finisher shown in Fig. 1a filled with 6.3 mm diameter steel balls. Its motion was determined using the accelerometer and laser displacement measurements as described in Ref. Hashemnia et al. (2013). The tub had sinusoidal translations in the *x* and *z* directions and sinusoidal rotation in the plane of Fig. 1b, all at 47 Hz. The equivalent material properties of the granular media needed in modeling the media as a continuum were estimated using grain-scale 3D DEM simulations.

The vibrationally-fluidized granular flow was then modeled as a continuum visco-plastic media using both Lagrangian and Eulerian finite element formulations (ABAQUS, 2014). The bulk flow velocities were compared with those obtained in DEM simulations and with experimental measurements made in the same tub finisher (Hashemnia et al., 2013; Hashemnia and Spelt, 2014).