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Temperature dependent viscosity and surface tension effects on deformations of non-isothermal falling liquid film

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ABSTRACT

Theoretical and numerical investigations of the heat transfer and hydrodynamics in a liquid film flowing along an inclined substrate under the action of gravity with a local heat source have been performed. A two-dimensional model, based on the thin layer approximation, has been developed describing deformations of the film interface. Equation of a non-isothermal thin-film flow with linear dependence of viscosity and surface tension on temperature is derived. A generalized analytical formula for the film thickness as a function of liquid flow-rate is obtained. Marangoni flow, due to local temperature changes, opposes the gravitationally driven film flow and forms a horizontal bump near the upper edge of the heater. Attention is paid to the viscosity effect on the shape of the bump and the film thinning on the local heaters. A second order deformation of the free surface before the bump up to flow may exist. The criterion for the appearance of this deformation is found analytically.

1. Introduction

Problems related to the improvement of heat transfer at interfaces between solids and flowing fluids have high technological interests. In case of a thin liquid film non-uniformly heated the action of Marangoni effect resulting from the large temperature gradients at the liquid–gas interface induces structural changes in the pattern of the flow that may lead to increasing or decreasing of the heat transfer coefficient and film rupture. The thermocapillary effects on gravitationally driven falling liquid film on a solid plate have been studied theoretically by Joo et al. [1], Kalliadasis et al. [2] and Miladinova et al. [3] for uniformly and non-uniformly heated plate, respectively. For a current review of the field see also paper by Oron et al. [4], as well as books by Alekseenko et al. [5] and Demekhin and Chang [6].

The onset of a horizontal liquid bump at the upper heater edge zone is common to all the experiments on thin films falling down non-uniformly heated plates [7–12]. Using the infrared thermography it is established in [7] that the formation of the bump has a thermocapillary nature. A region with maximum surface temperature gradients up to 15 K/mm appears in the bump zone. The experiments are performed on vertical and inclined plates with aqueous solution of ethyl alcohol (mass concentration 10% and 25%). The plate inclination angle varies from 4° to 90°. Deformation of the film as a bump at the top edge of the heater has been proved to exist since the smallest heat fluxes. With increasing the heat flux the height of the bump grows. The surface flow is made visible by aluminium tracers blown on the interface in [10].

In order to explain the horizontal bump and other phenomena in locally heated liquid films several models have been proposed in the papers [13–19] taking into account variations of surface tension with temperature. The temperature dependence of viscosity was neglected in these papers. In [9] a two-dimensional model has been studied taking into account variations of surface tension and viscosity with temperature. However numerical calculations were reported only for a heater with 6.7 mm streamwise length, as used in their experiments. The film is considered as thermal insulated on a free surface and on a substrate outside the heating element. On the heater the boundary condition for the heat flux $q(x)$ is set. A polynomial dependence of viscosity on temperature has been chosen. Therefore, the system of equations has not been reduced into a single equation for the film thickness. In [11] the numerical and experimental investigations of "lateral waves" are reported. Surface tension and viscosity are depending on temperature. Numerical calculations are executed for a heater with 6.7 mm streamwise length and 10% aqueous solution of ethyl alcohol only. In [20,21] the numerical and experimental investigations of regular structures formation in a film falling down a vertical plate with rectangular heater are reported. A polynomial dependence of viscosity on

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Nomenclature

A	dimensionless number ($\cos\Theta$/Fr)
Bi	Biot numbers (bH_0/k)
b	heat transfer coefficient, W/(m^2 K)
C	dimensionless number ($\sin\Theta/\varepsilon$Fr)
c_p	specific heat of the liquid, J/(kg K)
$\mathbf{D}$	velocity deformation tensor
Fr	Froude number (lU^2/gH_0^2)
G	dimensionless flow rate of the liquid
g	gravitational acceleration, m/s^2
H	local film thickness, m
h	dimensionless film thickness
Ka	Kapitza number, $((\rho\sigma_0^3/g\mu_0^4)^{1/3})$
k	thermal conductivity, W/(m K)
L	heater length, m
l	characteristic scale of streamwise length, m
Ma	Marangoni number ($\sigma_T[T]H_0/Ul\mu_0$)
Pr	Prandtl number ($\mu c_p/k$)
p	pressure, N/m^2
q	heat flux released on the heater, W/m^2
Re	Reynolds number ($\rho\Gamma/\mu$)
R_μ	Viscosity number, dimensionless ($\mu_T[T]/\mu_0$)
T	temperature, °C
$[T]$	characteristic scale of the temperature, K
U	characteristic scale of the liquid velocity, m/s
u, v	velocity components, m/s
x, y	Cartesian coordinates, m

Greek symbols

Γ	flow rate of the liquid per unit film width, m^2/s
$\delta(x)$	Dirac delta-function
Θ	plate inclination angle, °
θ	dimensionless temperature of the liquid
μ	liquid dynamic viscosity, kg/(m s)
ρ	liquid density, kg/m^3
σ	surface tension, N/m
χ	thermal diffusivity, m^2/s
$\chi(x)$	Heaviside function

Subscripts

0	initial parameters of the flow (at $T = T_0$)
2	gas phase
x,y,T	derivatives on x, y and T

Superscripts

+	limiting values ($x \to \infty$)
−	limiting values ($x \to -\infty$)

temperature is chosen. But influence of viscosity variations with temperature on rivulet structures formation has not been studied.

To avoid the reduction of performance of falling film evaporators by film breakdown it is of paramount importance to understand where and why the significant film thinning arises that may break the film. In the present paper a two-dimensional model is developed describing the deformations of the film interface. Surface tension and viscosity are assumed to depend on temperature linearly. The influences of the viscosity and its variation on the shape of the bump are carefully investigated. Also the second order deformation of the free surface before the bump has been investigated.

2. Liquids and properties

The main part of calculations is done for the solution of 25% (mass) ethyl alcohol in water. The dynamic viscosity and the surface tension are assumed to depend linearly on temperature: $\mu(T) = \mu_0 - \mu_T(T - T_0)$, μ_0, $\mu_T = const > 0$, $\sigma(T) = \sigma_0 - \sigma_T(T - T_0)$, σ_0, $\sigma_T = const > 0$. The linear approximation for dependence $\sigma(T)$ is correct within the uncertainty of the measurement of the surface tension in a wide range of temperatures [22,23].

The linear approximation for the $\mu(T)$ dependence is used, because for the temperature differences considered in the paper (less than 20 °C) the deviation of the dependence $\mu(T)$ from the linear one is not essential. The viscosity of the liquid is changed both across the film thickness and along the heater. For small Re numbers ($Re \leqslant 1$) the temperature differences across the film do not exceed 10 °C [10]. The bulk temperature difference of the film along the heater may be estimated as $T_F - T_0 = q_w L/(c_p \rho \Gamma)$, where T_F is the bulk temperature of the film at the end of the heater, Γ – flow rate of the liquid per unit film width, q_w – heat flux on the wall of the heater, L – heater length, ρ – liquid density, c_p – specific heat of the liquid. For the solution of 25% ethyl alcohol in water and the heater length 6.7 mm at $Re = 0.5$, the critical heat flux for rivulet structures formation is measured equal to $q_w = 1.21$ W/cm^2 [9]. Under these conditions a 15.4 °C temperature difference along the heater is calculated.

From temperature $T_0 = 20$ °C to temperature $T_F = 40$ °C the viscosity is reduced from $\mu = 2.45 \times 10^{-3}$ kg/(m s) to $\mu = 1.25 \times 10^{-3}$ kg/(m s) [23]. This variation of viscosity is as large as 96%. A linear dependence based on existing data is defined as $\mu(T) = (4.52122 - 0.101445 \times T) \times 0.001$ kg/(m s). The deviation of the tabulated data [23] from the linear dependence at the temperature range from $T_0 = 20$ °C to $T_F = 38$ °C is not higher than 6.5%. The thermal conductivity and the specific heat of the liquid in the same range of temperature variation increase by 7.7% and 0.26% respectively [23]. The values of the liquid properties at the initial fluid temperature 20 °C are: $\rho = 961.1$ kg/m^3, $k = 0.4781$ W/(m K), $\sigma_0 = 35.53 \times 10^{-3}$ N/m, $\sigma_T = 0.1103 \times 10^{-3}$ N/(m K), Pr = 22.06. Data for surface tension and surface tension variation with temperature obtained in [22] are used.

In the thin-layer approximation, as a rule, the simplified equations of the flow can be integrated exactly, and the problem reduces to solving one evolution equation for the film thickness [13]. For the temperature dependent viscosity, generally speaking, no exact integration of governing equations is possible [9,11]. This integration becomes possible if viscosity depends linearly on temperature. It also becomes possible to obtain a generalized analytical formula for the film thickness as a function of liquid flow-rate in non-isothermal thin liquid films.

3. Problem formulation

The two-dimensional steady-state deformations are studied for a non-isothermal thin film of viscous incompressible liquid, falling down a plate inclined at an angle Θ. Geometry of the problem is shown in Fig. 1. A Cartesian coordinate system (x, y) is chosen in order that the axis Oy is orthogonal to the substrate, and the axis Ox is directed along the liquid flow. It is known that a gravitational film flow at the vertical plane is unstable at any Reynolds numbers even without additional disturbing influences [5]. The falling film with a smooth interface exists on a starting part of the substrate (5–10 cm, [5,7,10,22]) or on the surface with inclination to the horizon plane smaller than some critical value [24]. It is supposed also that the length of the heating zone (typically 4–20 mm) is much smaller than the length of the smooth interface but much longer compare with the film thickness (0.1 mm).