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Stability analysis of stratified Rayleigh–Bénard–Poiseuille convection: Influence of the shear flow

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HIGHLIGHTS

- The effect of shear flow on stratified Rayleigh–Bérnard–Poiseulle convection has been studied.
- Linear stability analysis has been used to find the onset of natural convection.
- LSA results are compared and completed by performing numerical simulation with CFD techniques.

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ABSTRACT

The presence of horizontal flow in stratified systems where a vertical temperature gradient leads to heat transfer through natural convection can be observed in several technological and natural phenomena. In the study reported herein, a linear stability analysis using normal modes and the direct simulation of the governing equations using CFD techniques are applied to investigate the influence of the horizontal flow intensity on the onset of natural convection in a double-layer system heated from below. The results obtained with the two methodologies are in good agreement and complement each other, since, while linear analysis is suitable for defining the critical Rayleigh values, direct simulations allow a detailed analysis of the flow field when the convective motion is fully developed. Due to the large number of phenomena governing the system stability, this study focuses on a part of the spectrum of parameters selected to allow the determination of the minimum values that the Rayleigh number must achieve in order to make natural convection possible. The aim of the study is to gain a better understanding of some basic characteristics of the flow, such as the influence of the boundary conditions and the most common ways in which the convective cells can develop. The results are consistent with previously published data for double-layer Rayleigh–Bénard and single-layer Rayleigh–Bénard–Poiseuille convection. However, the existence of different modes that can make the system unstable creates a more complex scenario for some intervals of the governing parameters.

1. Introduction

The stability analysis of buoyancy-driven convection in superimposed layers of immiscible fluids has been extensively investigated in the past few decades, mainly due to the wide range of situations where this configuration appears in the fields of geophysics, astrophysics and engineering (Davaille, 1999; Bratsun and de Wit, 2011; Gubaidullin, 2003). Besides the natural convection generated by a vertical temperature gradient, in some situations the fluid motion can also be induced by a horizontal pressure gradient, creating a condition of mixed convection. Co-extrusion processes, oil–water flow and stratified atmospheric flow are some examples of multilayered systems in which this scenario can be found. In these systems, the flow configuration attained corresponds to stratified Rayleigh–Bénard–Poiseuille (*SRBP*) convection.

The SRBP convection display characteristics from both single-layer Rayleigh–Bénard–Poiseuille (*RBP*) and stratified Rayleigh–Bénard convection. Although the stability conditions of these two configurations are well known, the influence of the shear flow on the stability of multi-layer Rayleigh–Bénard convection is not yet completely understood, mostly because of the great number of governing parameters affecting the system stability.

The limits within which natural convection emerges as a result of specific perturbations are among the most valuable data provided by stability analysis, since they are associated with a substantial change

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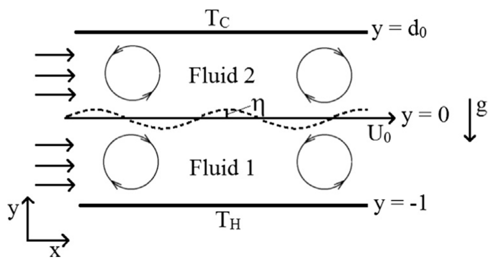

Fig. 1. Representation of the stratified Rayleigh–Bénard–Poiseuille convection showing the interface deformation η.

in the flow configuration. The stability analysis of single-layer Rayleigh–Bénard convection reveals a complex behavior at the onset of convection, such as multiplicity of stationary states and periodic regimes (see Chandrasekhar, 1961; Kao and Yang, 2007; Silano et al., 2010; Venturi et al., 2012), and as described by Andereck et al. (1998) the presence of more layers further increases the system complexity. In multi-layer systems different types of coupling between the layers can appear, depending on the intensity of the buoyancy forces in each layer and the interfacial forces (see Jhonson and Narayanan, 1997, Fig. 1, for the possible configurations).

When the buoyancy forces have comparable magnitude in the two layers (i.e., when the ratio between the Rayleigh numbers evaluated at each layer is close to 1), the coupling usually occurs as viscous or thermal coupling. In viscous coupling, adjacent cells in different layers have opposite directions of recirculation and so the vorticity at the interface has different signs in both layers, while in thermal coupling cells rotate in the same direction and the vorticity has the same sign. As presented by Rasenat et al. (1988), Cardin et al. (1991) and Colinet and Legros (1994), the competition between these coupling mechanisms can lead to an oscillatory state, even when the interface distortion is neglected. These periodic regimes appear as a result of a Hopf bifurcation and the oscillation frequency is a function of the wavenumber associated with the perturbation considered.

In addition to the buoyancy forces, thermal convective motion can arise due to the thermocapillary forces at the interface, as discussed by Zeren and Reynolds (1972). When the thermocapillary instability is dominant in systems of viscous fluids, the interface deformation can be significant and plays a key role in the system dynamics (Lebon et al., 2001). As shown by Bars and Davaille (2002, 2004), when the interfacial tension and the density stratification are not sufficient to maintain the interface stable, the system can evolve to states with high interface deformation, including situations where one layer penetrates the other and the convection takes place in the whole system as in a single-layer. Furthermore, as mentioned by Sun (2012), in several heat and mass transfer operations the system stability can be influenced by both thermocapillary and buoyancy forces.

In SRBP convection, the interface can also be distorted by the shear flow through the formation of interfacial waves that can lead to the system destabilization (see Boomkamp and Miesen, 1996 for the mechanisms of instabilities in stratified flows). In particular, the importance of viscosity stratification in the flow stability should be noted. Yih (1967) showed through an asymptotic analysis that for a stratified plane Poiseuille flow the discontinuity in the base flow caused by a difference in the viscosity of each layer can induce flow instability, even for very small Reynolds numbers. Yih's results were later expanded by Li (1969) to include a third fluid layer and by Yantsios and Higgins (1988) to evaluate perturbations with large wavenumber and differences in several governing parameters. Reddy et al. (2011) evaluated the effect of the wall-heating on the stability of a two-layer horizontal flow, neglecting the influence of the buoyancy forces. Using a linear stability analysis, the authors found a non-monotonic dependence between the growth rate of the most unstable mode and the temperature difference between the walls.

The mechanisms of instability in single-layer Rayleigh–Bénard–Poiseuille convection have been the subject of several studies in recent decades as well (see Nicolas, 2002 for a review of numerical and experimental studies). As discussed by Xin et al. (2006), in RBP convection the combination of Bénard cells and plane flow can create very complex structures even at low Rayleigh and Reynolds numbers. In a three-dimensional system, the convective rolls can develop in different ways, for example, as pure transverse or longitudinal rolls (Nicolas et al., 2000), superposition of these two modes (Cheng et al., 2002) and oblique rolls (Barletta and Nield, 2012). The continuous mass inflow and outflow allows the development of convective or absolute instability. As described by Grandjean (2008), convective instability occurs when a given perturbation grows and propagates in the flow direction, eventually leaving the system after a sufficiently long time. On the other hand, in absolute instability the perturbation grows and propagates in every direction and will over time fill the entire system. Details on the transition between convective and absolute instabilities in the RBP flow are provided by Carrière and Monkewitz (1999) and Grandjean and Monkewitz (2009).

From the above review of the literature it is apparent that there are several mechanisms that can make the SRBP convection system unstable. Therefore, to define the system stability many governing parameters have to be taken into account and thus a global analysis is difficult to be carry out. The aim of this paper is to investigate the changes in the flow structure in double-layer Rayleigh–Bénard convection due to the presence of a horizontal shear flow. Indeed, in many important engineering and geophysical flows, thermal convection occurs in the presence of mean shear flow (e.g. Domaradzki and Metcalfe, 1988).

The study will be carried out using two complementary methods: modal linear stability analysis (LSA) and numerical simulation of the governing equations using discretization methods. The former will be used to determine the onset of convection over a wide parameter range, whereas the latter can be applied to an in-depth investigation of the flow structure, including non-linear terms neglected in the linear analysis. The numerical simulation analysis is carried out through computational fluid dynamic (CFD) based techniques and will be here referred as 'direct simulation'. The main objective of this paper is to present the mathematical model used for the LSA and to introduce some basic characteristics of SRBP convection.

The paper is structured as follows: in Section 2 the governing equations and the mathematical formulation used for the LSA analysis are presented and the set of differential equations governing the system stability is obtained. In Section 3 the influence of the Reynolds number on the onset of natural convection is investigated using the LSA. The analysis by CFD simulations is presented and discussed in Section 4. Final remarks end the paper.

2. Mathematical formulation

A schematic representation of the physical domain is shown in Fig. 1. The system consists of two layers of immiscible fluids bounded by horizontal rigid walls and subjected to a constant pressure gradient which induces the flow in the x-direction. The system is assumed to be unlimited in the z- and x-directions. The thickness of the lower layer, d_1, is used as the length scale, so that the bottom wall is placed at the dimensionless position $y=-1$ and the upper wall is placed at $y=d_2/d_1=d_0$. Both walls are considered to be at constant fixed temperatures and only the case $T_H > T_C$ will be analyzed. The non-deformed interface is positioned at $y=0$ and the interface deformation is evaluated by the function $\eta=\eta(x,z,t)$. The horizontal velocity at the interface is