International Journal of Heat and Mass Transfer 55 (2012) 1355–1362

Contents lists available at SciVerse ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Investigation of combined heat and mass transfer by Lie group analysis with variable diffusivity taking into account hydrodynamic slip and thermal convective boundary conditions

M.A.A. Hamad [a,b,*], Md.J. Uddin [a], A.I.Md. Ismail [a]

[a] *School of Mathematical Sciences, Universiti Sains Malaysia, Penang 11800, Malaysia*
[b] *Mathematics Department, Faculty of Science, Assiut University, Assiut 71516, Egypt*

ARTICLE INFO

Article history:
Received 30 December 2010
Received in revised form 5 July 2011
Accepted 13 July 2011
Available online 31 October 2011

Keywords:
Hydrodynamic slip
Lie group
Moving plate
Combined heat and mass transfer
Thermal convective boundary condition

ABSTRACT

The present paper investigates heat and mass transfer over a moving porous plate with hydrodynamic slip and thermal convective boundary conditions and concentration dependent diffusivity. The similarity representation of the system of partial differential equations of the problem is obtained through Lie group analysis. The resulting equations are solved numerically by Maple with Runge–Kutta–Fehlberg fourth–fifth order method. A representative set of results for the physical problem is displayed to illustrate the influence of parameters (velocity slip parameter, convective heat transfer parameter, concentration diffusivity parameter, Prandtl number and Schmidt number) on the dimensionless axial velocity, temperature and concentration field as well as the wall shear stress, the rate of heat transfer and the rate of mass transfer. The analytical solutions for velocity and temperature are obtained. Very good agreements are found between the analytical and numerical results of the present paper with published results.

1. Introduction

Boundary layer flows over a flat plate have been extensively studied. For fluid flows in micro electro mechanical systems, the no slip condition at the solid–fluid interface must be replaced by slip condition [1]. The slip flow model proposes a relationship between the tangential components of the velocity at the surface and the velocity gradient normal to the surface [2]. The effect of linear slip $|u| = l\left|\frac{\partial u}{\partial y}\right|$, and nonlinear slip $|u| = l\left|\frac{\partial u}{\partial y}\right|^n$, where $l > 0$ is the slip length and $n > 0$ is a certain power parameter, on the hydrodynamic boundary layer over different geometries has been studied by many. This includes Martin and Boyd [3,4], Fang and Lee [5], Vedantam [6], Mattews and Hill [7,8], Wang [9], and Mukhopadhyay and Andersson [10]. A study by Thompson and Troian [11] indicates that the slip length is a nonlinear function of the shear rate. Abel and Mahesha [12] included the effect of variable thermal conductivity, heat source, radiation, buoyancy, magneto-hydrodynamic effects as well as viscoelastic behaviour of the fluid. Aziz [1] studied hydrodynamic and thermal slip flow boundary layers over a flat plate with constant heat flux boundary condition. He concluded that as the slip parameter increases, the slip velocity increases and the wall shear stress decreases. Recently, convective boundary condition has been applied by Aziz [13], Ishak [14], Makinde [15], Makinde and Olanrewaju [16], and Yao et al. [17]. The effect of slip flow and convective boundary condition with variable diffusivity on boundary layers has received limited attention and this motivates the present study.

As stated by previous researchers, heat and mass transfer from different geometries has many engineering and geophysical applications. Examples include geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed bed catalytic reactors and underground energy transport. Coupled heat and mass transfer problems in the presence of chemical reactions are important in many processes and have, therefore, received attention of investigators in recent years [18].

As stated by Willbanks [19], Crank [20], and Azuara et al. [21], the diffusion coefficient depends only on the concentration of diffusing substance. Such concentration dependence exists in most systems, but generally the dependence is only slight and the diffusion coefficient can be assumed to be constant. However, there are some problems where the diffusion coefficient varies with the concentration over a certain range. The diffusion coefficient can often be approximated by a linear or exponential dependence equation.

Lie group analysis can be used to generate similarity transformations. It reduces the number of independent variable of the partial differential equations under consideration and keeps the system and associated initial and boundary condition invariant. This technique has been applied by many researchers to solve

* Corresponding author at: Mathematics Department, Faculty of Science, Assiut University, Assiut 71516, Egypt. Tel.: +20 183226444.

E-mail address: m_hamad@aun.edu.eg (M.A.A. Hamad).

0017-9310/$ - see front matter © 2011 Published by Elsevier Ltd.
doi:10.1016/j.ijheatmasstransfer.2011.08.043

different flow phenomena over different geometries such as Ibrahim et al. [22,23], Pandey et al. [24], Jalil et al. [25], and Hamad and Ismail [26]. Reviews for the fundamental theory and applications of Lie group analysis to differential equations may be found in the texts by Olver [27], Bluman and Kumei [28], Hill [29], Cantwell [30], and Ibragimov and Kovalev [31].

The aim of our present investigation is to find new similarity transformations and corresponding similarity solutions and to investigate heat and mass transfer of a Newtonian fluid with slip and convective boundary conditions taking into account the concentration dependent thermal diffusivity. Also,we study the influence parameters, namely, velocity slip parameter, convective heat transfer parameter and concentration diffusivity parameter on the dependent similarity variables.

2. Mathematical formulation of the problem

Consider a two dimensional steady viscous incompressible flow over a moving porous plate as shown in Fig. 1(i–iii, respectively represent concentration, thermal and momentum boundary layers). The field variables are the velocity components $\bar{u}, \bar{v}$ and the temperature T and the concentration C. The left surface of the plate is heated by convection from a hot fluid of temperature T_f which provides a heat transfer coefficient h_f. The governing boundary layer equations in dimensional forms are [32]

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}, \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}$$

$$\bar{u}\frac{\partial C}{\partial \bar{x}} + \bar{v}\frac{\partial C}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}}\left(D(C)\frac{\partial C}{\partial \bar{y}}\right). \tag{4}$$

The boundary conditions are taken as [13]

$$\begin{aligned} &\bar{u} = \bar{u}_w(\bar{x}) + N_1\nu\frac{\partial \bar{u}}{\partial \bar{y}}, \quad \bar{v} = -v_w, \quad -\kappa\frac{\partial T}{\partial \bar{y}} = h_f[T_f - T_w],\\ &C = C_w \quad \text{at} \quad \bar{y} = 0,\\ &\bar{u} \to 0, T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad \bar{y} \to \infty, \end{aligned} \tag{5}$$

where, ν: the kinematic coefficient of viscosity, α: thermal diffusivity, N_1: the velocity slip factor, $D(C)$: variable diffusivity, $\bar{u}_w(\bar{x})$: velocity of the moving plate, v_w: transpiration velocity ($v_w < 0$ for injection, $v_w > 0$ for suction), κ: thermal conductivity, T_w: wall temperature. The subscripts w, ∞ denote wall conditions and free stream conditions respectively. Here we assume that the dimensions of N_1 is $(\text{velocity})^{-1}$.

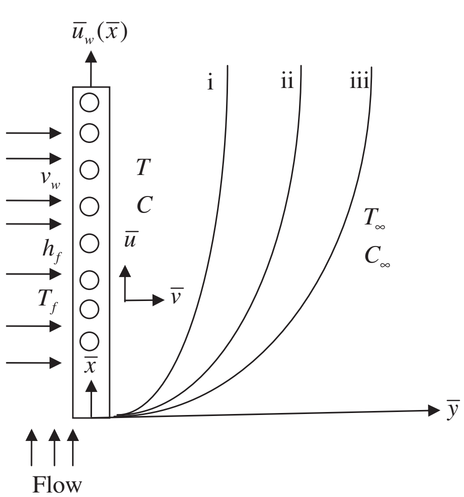

Fig. 1. Physical configuration and coordinate system of the problem.

3. Nondimensionalization of the governing equations

Introducing the following dimensionless variables

$$\begin{aligned} &x = \frac{\bar{x}}{L}, \quad y = \frac{\bar{y}}{L}\sqrt{Re},\\ &u = \frac{\bar{u}}{U}, \quad v = \frac{\bar{v}}{U}\sqrt{Re},\\ &\theta = \frac{T - T_\infty}{T_f - T_\infty},\\ &\phi = \frac{C - C_\infty}{C_w - C_\infty} \quad \text{and} \quad u_w = \frac{\bar{u}_w(\bar{x})}{U} \end{aligned} \tag{6}$$

with L being characteristic length, U is some reference velocity and $Re = \frac{UL}{\nu}$ is the Reynolds number.

The dimensionless forms of Eqs. (1)–(5) are as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2}, \tag{8}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \Pr^{-1}\frac{\partial^2 \theta}{\partial y^2}, \tag{9}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = Sc^{-1}\frac{\partial}{\partial y}\left((1 + b\phi)\frac{\partial \phi}{\partial y}\right). \tag{10}$$

The dimensionless boundary conditions are

$$\begin{aligned} &u = u_w(x) + a\frac{\partial u}{\partial y}, \quad v = -fw, \quad \frac{\partial \theta}{\partial y} = -\gamma(1 - \theta), \quad \phi = 1 \quad \text{at} \quad y = 0,\\ &u \to 0, \quad \theta \to 0, \quad \phi \to 0, \quad \text{as} \quad y \to \infty. \end{aligned} \tag{11}$$

Here $a = \frac{N_1\nu}{L}\sqrt{Re}$, $\gamma = \frac{h_f L}{k\sqrt{Re}}$, $b = c(C_w - C_\infty)$, $fw = \frac{v_w}{U}\sqrt{Re}$, $\Pr = \frac{\nu}{\alpha}$ and $Sc = \frac{\nu}{D}$.

The quantities a, γ, b, Pr and Sc are respectively hydrodynamic slip parameter, convective heat transfer parameter, concentration diffusivity parameter, Prandtl number and Schmidt number respectively. Here we have assumed that concentration diffusivity varying linearly and is given by White and Subramanian [33]

$$D(C) = D_m[1 + c(C - C_\infty)] = D_m[1 + b\phi], \tag{12}$$

where D_m is constant concentration diffusivity and c is a constant.

Introducing stream function ψ defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \tag{13}$$

we get from Eqs. (8)–(10),

$$\frac{\partial^3 \psi}{\partial y^3} + \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} = 0, \tag{14}$$

$$\frac{\partial^2 \theta}{\partial y^2} + \Pr\frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} - \Pr\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} = 0, \tag{15}$$

$$\frac{\partial}{\partial y}\left[(1 + b\phi)\frac{\partial \phi}{\partial y}\right] + Sc\frac{\partial \psi}{\partial x}\frac{\partial \phi}{\partial y} - Sc\frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial x} = 0. \tag{16}$$

The corresponding boundary conditions in Eq. (11) become