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Numerical modelling of three-fluid flow using the level-set method

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HIGHLIGHTS

- A model for simulation of three-fluid flow with two moving interfaces is proposed.
- Two level-set functions are used to capture the two interfaces for three-fluid flow.
- Simulation of three-fluid stratified flow is demonstrated based on the model.
- Two-fluid stratified flow with a third fluid in the form of drops are simulated.
- Simultaneous rising and settling of two drops in a third fluid are simulated.

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ABSTRACT

This paper presents a numerical model for simulation of three-fluid flow involving two different moving interfaces. These interfaces are captured using the level-set method via two different level-set functions. A combined formulation with only one set of conservation equations for the whole physical domain, consisting of three different immiscible fluids, is employed. Numerical solution is performed on a fixed mesh using the finite volume method. Surface tension effect is incorporated using the continuum surface force model. Validation of the present model is made against available results for stratified flow and rising bubble in a container with a free surface. Applications of the present model are demonstrated via a variety of three-fluid flow systems including (1) three-fluid stratified flow, (2) two-fluid stratified flow carrying the third fluid in the form of drops and (3) simultaneous rising and settling of two drops in a third stationary fluid.

1. Introduction

Three-fluid flow is widely encountered in the petroleum industry (Thorn et al., 1999, 2013). A good understanding of three-fluid flow is of practical importance, e.g. flow of water, oil and gas in the oil transportation pipelines. In these systems, the three fluids are generally separated by interfaces. These interfaces determine the flow pattern which is intimately related to the pressure drop and gas holdup in the pipes. Therefore, knowing the distribution of the three fluids in the pipes is important for the safety operation of the systems.

The flow pattern in pipes can be visualized experimentally (Açikgöz et al., 1992; Wu et al., 2001; Spedding et al., 2007; Xu et al., 2012). However, experiments generally involve building test rig and employing high resolution equipment for reliable data collection, i.e. economically demanding. Analytical study and numerical simulation therefore play essential complementary role in this respect. Analytical study of three-fluid is only possible for limited cases given the complexity of the flow systems (Taitel, et al., 1995; Ghorai et al., 2005). Most of the results are obtained with many simplifications made in the models. On the other hand, with numerical simulation, some of these restrictions can be removed and therefore incorporate more desirable flow physics into the model.

Numerical simulation of three-fluid flow can be categorized into two groups based on the ways the fluids are handled. These are continuous fluid method and discrete fluid method. The continuous fluid method is developed primarily to compute homogeneous flows. In this method, the fluids are treated as a mixture component which shares the same pressure field. The average velocity based on the volume fraction of each fluid is chosen to represent the information for all the phases. The methods include, but are not limited to, the two-fluid models and the mixture model. The discrete fluid method handles all the fluids separately as individual phase with distinct interfaces separating each other. Each fluid can then be modeled with their

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individual physical properties. All the fluids are characterized by their own velocity and pressure fields. Central to this method is to capture the interfaces between the different fluids. These interfaces could be tracked explicitly using a front-tracking approach, e.g. moving mesh method (Quan and Schmidt, 2006) or captured implicitly via a front-capturing approach, e.g. volume of fluid method (Hirt and Nichols, 1981) and the level-set method (Osher and Fedkiw, 2001).

Existing numerical works on three-fluid flow are fewer compared to that of two-fluid flow. Bonizzi and Issa (2003) developed a mathematical model to predict three-fluid slug flow in a nearly horizontal pipe using the two-fluid model. The relative motion of the fluid is considered using drift-flux model. Their results showed that the slip velocity between the liquid fluids is important. Cazarez et al. (2010) proposed a one-dimensional model based on the two-fluid model to simulate water-oil-gas flow in a vertical pipe. With drag and virtual mass forces in their models accounted for, their results are in good agreement with experimental data. Although the two-fluid model can be applied in many multiphase systems to predict the pressure drop and gas holdup, it however, produces poor solutions in the situation where the fluids interact and affect each other significantly. Under such a condition, the discrete fluid method is more desirable in revealing the detailed insight of fluid distribution as well as the interactions among different fluids. Pan and Chang (2000) used a surface capturing method (Kelecy and Pletcher, 1997) to investigate three-fluid flow in a container. The distribution of the three fluids as well as the two interfaces separating the three phases is well captured. However, surface tension force is not included in their simulation. Tofighi and Yildiz (2013) proposed a new surface tension model to study three-fluid flow using the smoothed particle hydrodynamics (SPH) method (Monaghan, 1994). The capability of their model is showcased for different flow problems including droplet levitation and droplet spreading. Reasonable agreement is achieved when compared against the existing analytical solutions. A moving mesh method to deal with the triple junction point formed by three-fluid interfaces is proposed by Li (2013). The boundary of the mesh is used to represent the interfaces. The method is applied to simulate the rising of a liquid droplet with an attached bubble in a lighter liquid.

Three-fluid flow generally can have two or three interfaces temporally evolving. The case with three interfaces is rather complex as it involves treatment of triple junction point. Only a few works have been reported so far on three-fluid flow with three interfaces (Chang et al., 1996; Smith et al., 2004; Yang and Ma, 2007; Starinshak et al., 2014). Chang et al. (1996) developed a general level-set formulation which can be used to study the interaction of two bubbles with different densities in a third immiscible liquid. The coalescence of these two bubbles was simulated. Smith et al. (2004) simulated a drop lying on the two-fluid interface in a shear flow. Three level-set functions were used to capture the triple junction of the three fluids. They found that a critical capillary number governs the shape of the drop. Yang and Ma (2007) simulated a moving drop covered by a layer of vapor in a water pool. The deformation of the vapor layer is studied. Starinshak et al. (2014) proposed a new level-set model to simulate multi-fluid flows. Their model is more general as it can be used to treat three-dimensional multi-fluid flow. These models are generally complex as they involve the treatment of the triple junction occurring in the three-fluid flow. In many engineering applications, three-fluid flow with two interfaces is widely encountered, e.g. three-fluid slug flow and three-fluid stratified flow (Xu et al., 2012). Interestingly and to the best knowledge of the authors, three-fluid flow with two interfaces under the frame work of level-set method has not received due attentions. The current work therefore focuses at this type of three-fluid flow. A level-set method with two level-set functions to treat the two interfaces is presented in this article. The capability of the current model is demonstrated for three-fluid stratified flow, two-fluid stratified flow with a third immiscible drop, simultaneous rising and settling of two drops in a third stationary fluid. It is necessary to mention that the aim of the present study is not to delve deeply into the rich physics involved in these three-fluid flows, but rather to demonstrate a mean to facilitate the investigation of such flows.

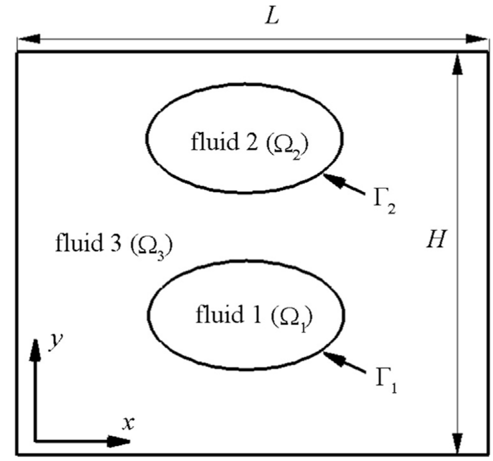

Fig. 1. Domain of interest with three fluids separated by two interfaces.

2. Problem description

The domain of interest is schematically shown in Fig. 1. It consists of three sub-regions, i.e., fluid 1 region (Ω_1), fluid 2 region (Ω_2) and fluid 3 region (Ω_3). The three fluids are immiscible in each other. These three regions are separated by interfaces Γ_1 and Γ_2. When the three fluids flow, the two interfaces evolve dynamically with time. The motions of the three fluids and the interfaces are inter-related and strongly coupled. It is necessary to mention fluid 1 and fluid 2 are always separated by fluid 3 in the current work.

3. Mathematical formulation

3.1. Governing equations

In this study, the level-set method is used to capture the evolving interfaces (Osher and Sethian, 1988). The level-set function $\phi\left(\vec{x},t\right)$ is defined as the signed shortest distance from the interface. With two interfaces for the three fluids shown in Fig. 1, i.e. Γ_1 and Γ_2, two level-set functions are required to represent these interfaces. These level-set functions are defined respectively as

$$\phi_1 \equiv \begin{cases} -d_1, & \text{if } \vec{x} \in \Omega_2 \cup \Omega_3 \\ 0 & \text{if } \vec{x} \in \Gamma_1 \\ +d_1, & \text{if } \vec{x} \in \Omega_1 \end{cases} \tag{1}$$

$$\phi_2 \equiv \begin{cases} -d_2, & \text{if } \vec{x} \in \Omega_1 \cup \Omega_3 \\ 0 & \text{if } \vec{x} \in \Gamma_2 \\ +d_2, & \text{if } \vec{x} \in \Omega_2 \end{cases} \tag{2}$$

where d_1 and $d_2$are the shortest distance to the interfaces $\Gamma_1$and Γ_2, respectively. The movements of the interfaces are captured by monitoring the evolution of the level-set functions governed by