Chemical Engineering Science 126 (2015) 471–482

Contents lists available at ScienceDirect

Chemical Engineering Science

journal homepage: www.elsevier.com/locate/ces

Multi-scale permeability of deformable fibrous porous media

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HIGHLIGHTS

- A numerical model for fluid-induced topology in fibrous porous media is proposed.
- Topology is defined by clustering, computed as convolution of fiber distribution.
- 2D clustering locally depends on porosity, shear rate and out-of-plane forces.
- Permeability may decrease one order of magnitude due to flow-induced variations.
- A hybrid FV/LB CFD method is developed to solve the multiscale fluid flow problem.

ARTICLE INFO

Article history:
Received 20 August 2014
Received in revised form
11 November 2014
Accepted 15 November 2014
Available online 22 December 2014

Keywords:
Resin transfer moulding (RTM)
Transport properties
Fabrics/textiles
Fibers
Multiscale modeling

ABSTRACT

The contribution of fiber dynamics and clustering to the effective permeability in hierarchical fibrous media is poorly understood, due to the complex fluid–structure interactions taking place across fiber, yarn and textile scales. In this work, a two-dimensional model for fiber deformation subject to out-of-plane movement restrictions is derived for creeping flow conditions by analogy with non-Brownian suspensions of particles with confining potentials. This leads to a homogeneous Fokker–Planck equation in a phase space of fiber configurations, for the probability density function of the fiber displacements. A fiber clustering criterion is then defined using autoconvolution functions of the local probability densities, which yields the local change in fiber-scale permeability according to a topological description of the porous media instead of the typical geometric description. The resulting multi-scale hydrodynamic system is numerically solved by a coupled method, where the Stokes flow at yarn-scale is solved with a finite volume method and the mesoscopic model that recovers information from the fiber-scale is solved by a lattice Boltzmann method. The fiber-scale permeability is characterized in terms of porosity, dimensionless shear rate and dimensionless out-of-plane forces. When assessed in terms of a reduced viscosity related to Brinkman's closure for porous media, the mesoscopic model shows that deformable fibrous porous media qualitatively behave like dense particle suspensions. For low volume fractions a non-Newtonian reduced viscosity exhibiting shear-thinning and low- and high-shear *plateaux* is obtained. For high volume fractions and high shear rates the out-of-plane forces lead to shear thickening. The results on steady fiber-scale permeability are presented in the form of phase diagrams which show that in the typical range of parameters for textiles, the effective permeability of the deformable case can be up 60% lower than that of the rigid case due to the formation of fiber clusters.

1. Introduction

Accurate permeability prediction of textile preforms is remarkably important for the manufacturing of fiber-reinforced thermosetting composites (Potter, 1997). The final quality of the component mostly depends on process variables, whose optimization require the textile permeability as an input parameter. Textile preforms for these applications generally present a hierarchical structure and therefore different length scales to be taken into account, typically ranging between one and three orders of magnitude. As a consequence, the numerical solution of the fluid flow in the real geometry is computationally expensive or even not affordable with standard techniques when length scales diverge. A common practice consists in separating scales and/or reducing the dimensionality of the problem. This allows for the so-called "constitutive" relations (Hunt et al., 2014), that is, analytical solutions or experimental correlations which serve as auxiliary means for the numerical simulations. A review of the several experimental and analytical techniques developed in this framework

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http://dx.doi.org/10.1016/j.ces.2014.11.065
0009-2509/

can be found in Åström et al. (1992) and Yazdchi and Luding (2012). One of the best established correlations was proposed by Gebart (1992), who derived analytically a permeability law for ordered arrangements of cylinders:

$$\frac{K}{R^2} = C\left[\sqrt{\frac{1-\varepsilon_c}{1-\varepsilon}} - 1\right]^{5/2}, \tag{1}$$

where K is the permeability, R the characteristic radius, C a geometrical factor depending on the arrangement, ε the porosity and ε_c its critical value (or percolation threshold). The author calculated $C = 16/9\pi\sqrt{2}$, $\varepsilon_c = 1-\pi/4$ for square arrangements and $C = 16/9\pi\sqrt{6}$, $\varepsilon_c = 1-\pi/2\sqrt{3}$ for hexagonal ones. Papathanasiou (1997) addressed the multi-scale nature of the problem by solving numerically a square array layout of permeable multi-filament yarns with circular fibers and showed that the effective permeability depends strongly on the microscopic porosity only at low values of the macroscopic one. He proposed a dimensionless correlation for the multi-scale permeability in the form (Papathanasiou, 2001):

$$K_{\text{eff}} = K_M\left[1 + a_1\left(\frac{K_M}{K_m}\right)^{n-3/2}\right], \tag{2}$$

where K_M and K_m correspond to the yarn (macro) and fiber (micro) permeabilities, respectively. The constants a_1 and n are geometrical parameters that the author best fitted with numerical simulation data, obtaining: $a_1 = 2.3$ and $n = 0.59$ for square arrangements, $a_1 = 3.0$ and $n = 0.625$ for hexagonal. Analogous studies were conducted for different yarn cross-sections and arrangements (Markicevic and Papathanasiou, 2002; Papathanasiou et al., 2002). Due to the geometrical dependence on the percolation threshold, the validity of the above (or similar) correlations is limited to strictly regular layouts, both at the macro- and microscopic scales. Consequently, their use for the numerical simulation of textile geometries often results in an unacceptable loss of accuracy due to: (i) the false assumption of regular topologies and (ii) the deformation of the structures induced by the fluid flow.

In order to overcome the first issue (i), random or realistically reconstructed fiber configurations have been extensively studied (Endruweit et al., 2013; Soltani et al., 2014) and statistical descriptors have been proposed to relate the permeability to non-regular fiber arrangements (Chen and Papathanasiou, 2008; Yazdchi et al., 2012). The effect of several micro-structural parameters on the effective permeability has been also investigated using up-scaling techniques (Yazdchi et al., 2011; Yazdchi and Luding, 2013). However, despite the intense work on configurations and up-scaling, the fluid–structure interaction problem (ii) has not been addressed in this framework, as far as the present authors know.

The flow-induced deformation of fibers, however, affects the interconnectivity of the porous matrix and thus the percolating paths, which in turn affect the permeability (Hunt et al., 2014). Indeed, relevant recent work on the permeability of deforming porous matrices relies on the idea that the flow resistance of particle clusters (in two dimensions) is larger than that justifiable by single particle contributions. This is basically due to the entrapment of fluid within the cluster, which increases the apparent volume fraction reducing the hydraulic (or wet) area. Scholz et al. (2012) recently proposed a generalized empirical expression for permeability based on this concept:

$$K = c l_c^2\left(\frac{1-\chi_o}{N}\right)^{\beta}, \tag{3}$$

where c is a constant that depends on the local pore geometry, l_c is the limiting hydrodynamic length, χ_o is the open-space Euler characteristic (of the conducting phase), N is the number of particles and β is the conductivity exponent. The Euler characteristic χ is a Minkowski functional that in this framework is defined as the difference between the number of connected components of each phase (Mecke and Arns, 2005); thus χ_o is the difference between the liquid phase and the number of solid components (neglecting the fluid entrapped in closed cavities). The authors best fitted $\beta = 1.27$ against experimental and numerical data for quasi-two-dimensional porous structures (close to the critical value $\beta_c = 1.3$ for two-dimensional structures Scholz et al., 2012). The quantity $1-\chi_o$ is generally referred to as *genus* and represents the total number of clusters of single or touching particles; thus $(1-\chi_o)/N$ is the number of clusters per particle or *cluster density*, which in the following will be called Ω for compactness.

Based on this latter idea, in this work we propose a multi-scale framework for the analysis of the local fiber topology induced by the fluid flow, through the cluster density. A two-dimensional mesoscopic model for the deformation of fibers subject to out-of-plane movement restrictions is derived for creeping flow conditions by analogy with non-Brownian systems with confining potentials. Typical non-Brownian examples are the suspensions of particles, such as rods or short fibers in a polymer (for composite manufacturing) and also the pulp in paper manufacturing (Larson, 1998). The difference of the proposed model and these non-Brownian examples is that we use out-of-plane forces, instead of long-range forces/potentials. The mesoscopic model is a homogeneous Fokker–Planck equation in a phase space of fiber configurations, for the probability density function of the fiber displacements. A fiber clustering criterion is then defined via autoconvolution functions of the probability densities, which yields the local topology of the fibers and the related change in permeability through the cluster density Ω. The resulting multi-scale hydrodynamic system is solved numerically by a coupled finite-volume/lattice-Boltzmann method (the latter accelerated on graphic processing units). Due to the lack of experimental or analytical means for its validation, the behavior of the proposed model is assessed in terms of a non-Newtonian reduced viscosity related to the Brinkman closure for porous media. The resulting rheology shows qualitative agreement with that of wet granular media.

The work is organized as follows. The theoretical model is explained in Section 2: firstly, the modeling framework is introduced, then the macroscopic equations are derived from the microscopic scale via volume-averaging technique and the models for the fiber dynamics and clustering are detailed. A very brief description of the numerical methods follows in Section 3. Section 4 is dedicated to the introduction and discussion of the results, which comprehend: a parametric analysis of the proposed model; the assessment of its behavior in terms of the reduced viscosity and the results obtained for the permeability of multi-scale fibrous media. The conclusions and an outlook on further work conclude the paper (Section 5).

2. Theoretical model

Let us consider the modeling framework shown in Fig. 1. We consider two scales: a macro-scale (the yarn scale in Fig. 1(a)) and a micro-scale (the fiber scale in Fig. 1(b)). The relative two-dimensional representative elementary volumes (REV) are shown on the right, respectively REV_M and REV_m. A macroscopic porosity ε_M is defined as the void fraction in REV_M and a microscopic one ε_m as the void fraction in REV_m.

The fibers are assumed to be clamped at both ends, thus they can bend under the effect of the perpendicular flow field. The cross-section of the yarn results in a domain of two-dimensional interacting particles suspended in the fluid (Fig. 1(a)), whose movement is restricted by the out-of-plane constraint.

Each fiber can bend up to ξ_{max}, which is a function of the distance z from REV_m to the clamped end (Fig. 1(b)). The length of