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Nonlinear simulations of miscible viscous fingering with gradient stresses in porous media

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HIGHLIGHTS

- Non-linear simulations of VF with the Korteweg stresses effect.
- Long time behavior of such Korteweg stresses.
- Korteweg stress stabilizes the downstream fingers more than the upstream fingers.
- Korteweg stress seems to act against the broadening of the fingertip.

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ABSTRACT

Long-time behavior of the nonlinear simulations of miscible viscous fingering (VF), which arises during the displacement of a high viscous fluid by a lesser viscous one in a porous media, has been investigated in the presence of gradient stresses. Such non-conventional stresses appear in a miscible fluid system having steep concentration, density or temperature gradient. Experiments show that these gradient stresses, also called the Korteweg stresses, cause an effective interfacial tension (EIT) and act for the stabilization of the system against the growth of the fingers. Such fluid flow systems have been modeled by coupling the Darcy–Korteweg equation with the convection–diffusion equation for the evolution of the solute concentration. These equations are solved simultaneously using a highly accurate Fourier spectral method. Investigations have been carried out for classical single interface VF instability and it has been shown that the Korteweg stress stabilizes the downstream fingers more than the upstream fingers. The propagation of the non-zero axial velocity field, which is spread away from the fingertips, explains this effect. Korteweg stress seems to act against the broadening of the fingertip that resists the splitting of an isolated finger. The growth rate of the unstable modes at an early time of the nonlinear simulations is obtained for various flow parameters and the results obtained are found to be qualitatively in good agreement with the linear stability results available in the literature.

1. Introduction

Viscous fingering instability plays an important role in displacement processes in porous media. It is a pervasive hydrodynamic interfacial instability that appears due to the displacement of a more viscous fluid by a less viscous one flowing through porous materials (Homsy, 1987). This instability phenomenon has drawn attention of numerous researchers from various science and engineering communities over the past several decades. Extensive studies have been conducted for various industrial and environmental processes such as secondary and tertiary oil recovery in oil industries, hydrology, filtration, chromatographic column (De Wit et al., 2005; Mishra et al., 2008; Rousseaux et al., 2007; Shalliker et al., 2007), fixed bed regeneration, aquifers (De Wit et al., 2005) and geodynamics (Morra and Yuen, 2008). VF has also been used as an alternative source to enhance mixing of two fluids in micro-channel in low Reynolds number regime (Jha et al., 2011), where turbulence does not appear. Although the miscible and immiscible fluids are of two completely different categories, they encounter qualitatively similar VF instability except the stabilizing phenomena in the respective cases. In case of miscible fluid, diffusion process acts for the stabilization, whereas for immiscible VF surface tension acts against the growth of the fingers and tries to stabilize the system.

However, in the case of miscible fluid displacement, a sharp concentration, density or temperature gradient can result in a

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non-conventional stress in the system that mimics the surface tension effect in miscible fluids. Such stress, called the Korteweg stress (Korteweg, 1901), acts at the miscible diffusive interface against the growth of the instabilities (Pojman et al., 2007, 2006, 2009; Joseph, 1990; Hu and Joseph, 1992; Chen and Meiburg, 2002; Pramanik and Mishra, 2013, 2014; Swernath et al., 2010). Experiments are conducted with the miscible fluids in microgravity to determine if they could exhibit transient interfacial phenomena seen with immiscible fluids (Pojman et al., 2007). Neither Rayleigh–Tomotika instability appeared for a stream of honey injected into water nor a blob of water changed its shape during the experimental run. Numerical simulation with the Korteweg stresses exhibited that the dimensional gradient stress parameter should be $> 10^{-12}$ N for measurable change in the shape of the asymmetric drop. In another experiment Pojman et al. (2006) observed the existence of an effective interfacial tension and occurrence of Rayleigh–Tomotika instability for the miscible mixture of isobutyric acid and water above the upper critical solution temperature. Similar phenomena were noticed for 1-butanol–water mixture below the solubility limit.

According to these experimental evidences modeling for the convection of miscible fluids system should incorporate the gradient stress term arising from the non-local interactions in the fluids. Although the proposal of such stress term was given more than a century ago by Korteweg (1901), who first presented a constitutive equation including the stresses induced by gradients of compositions, it is required to be studied extensively in several applications where steep gradient may appear. Almost after 90 years Joseph (1990) reconsidered the equations in order to explain the deformation of a rising water bubble in glycerin. He was successful to conclude that the gradient stresses mimic a surface tensional effect which helps the water drop to become spherical with a very narrow tail. Hu and Joseph (1992) investigated the displacement of miscible fluids of different densities in the Hele–Shaw cell with gradient stresses and accomplished the similarities of the gradient stresses with those of interfacial tension in immiscible fluids.

Notable discrepancies are recognized for the experimental findings of miscible fluids displacement in capillary tubes of Petitijeans and Maxworthy (1996) with the numerical simulations of miscible fluids in capillary tubes following the Stokes equation (Chen and Meiburg, 1996). In order to demonstrate the physical reasoning behind these discrepancies Chen and Meiburg (2002) followed the model of Joseph and co-authors. It is perceived that the density difference between the two fluids has very negligible contributions compared to the gradient stresses in the observed discrepancies. Recently, Pramanik and Mishra (2013) have investigated the onset of VF for classical single interface displacement with or without the Korteweg stresses effect through a linear stability analysis in similarity transformation domain. A delay at the onset of the VF instability was captured when the Korteweg stresses act at the miscible interface of the underlying fluids. Most recently, Truzzolillo et al. (2014) have performed Hele–Shaw experiments with miscible colloidal suspensions that confirm the stabilization property of the Korteweg stresses on the miscible VF in the Hele–Shaw cell or 2D porous media. They have shown that the diffusion rate of the colloidal suspension is very slow such that the relaxation time window is large enough to experimentally observe the effect of the Korteweg stresses.

Discussion of the above-mentioned literature reveals that in the presence of sharp compositional gradients between the fluids a non-conventional stress acts in miscible fluid systems. Thus, although the stability analyses of miscible and immiscible fluids are two completely different problems, one can attempt to understand them in a unified manner. According to the authors' knowledge the influence of gradient stresses on the nonlinear interactions in miscible VF of classical single interface displacement is not known in the literature, and it has been studied in this paper. The long time behavior of the Korteweg stresses on the fundamental features of VF instability such as splitting, merging, coarsening, etc. has been analyzed in this present study. The obtained results are compared with those obtained from the linear stability analysis.

It is to be noted that the Korteweg stress has been addressed as a gradient stress since it appears due to steep compositional gradient. Sometimes, it is also called as transient interfacial tension or effective interfacial tension (EIT). However, in this present study we shall mostly use the terminology Korteweg stress or gradient stress. The paper is organized in the following manner. In Section 2 the problem has been described mathematically along with the initial, boundary conditions and the numerical method used for solving the problem. Obtained results for single interface case have been analyzed in Section 3. In Section 4 the obtained numerical results are compared with the linear stability analysis. Concluding remarks are presented in Section 5.

2. Mathematical formulation and its solution

2.1. Formulation

We consider the displacement of miscible fluids having a steep concentration gradient in a medium with uniform porosity and constant permeability, k, as shown in Fig. 1. Initially the domain is filled with two fluids of different viscosities. The viscosity of the fluid region $x > 0$ is μ_2 and that of the fluid region $x < 0$ is μ_1. The latter fluid enters into the porous medium or the Hele–Shaw cell with a uniform velocity U and displaces the former. Fluids are assumed to be incompressible and neutrally buoyant to avoid the density gradient instability. The viscosity, $\mu(c)$, of the fluids depends strongly on the concentration of the sample solvent. The sample solvent concentration is $c=0$ and $c=c_2$ in the displacing and displaced fluids, respectively.

2.2. Governing equations

The problem has been modeled by coupling convection–diffusion equation for the sample solvent with the Darcy–Korteweg equation (Pramanik and Mishra, 2013) for fluid flow velocity. The model described above can be governed by the following equation (Joseph and Renardy, 1992; Pramanik and Mishra, 2013, 2014; Swernath et al., 2010):

$$\nabla \cdot \vec{u} = 0, \quad (1)$$

$$\nabla P = -\frac{\mu}{k}\vec{u} + \nabla \cdot [\hat{\delta}(\nabla c)(\nabla c)], \quad (2)$$

$$\frac{\partial c}{\partial t} + \vec{u} \cdot \nabla c = D\nabla^2 c, \quad (3)$$

$$\mu = \mu(c). \quad (4)$$

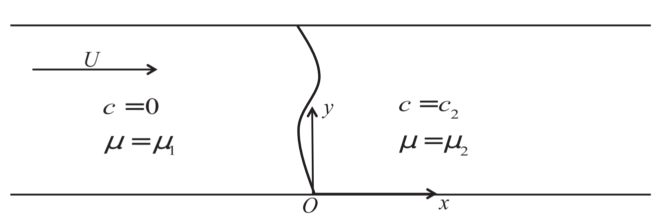

Fig. 1. Schematic diagram of displacement of a miscible fluid by another fluid in a Hele–Shaw cell.