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General pure convection residence time distribution theory of fully developed laminar flows in straight planar and axisymmetric channels

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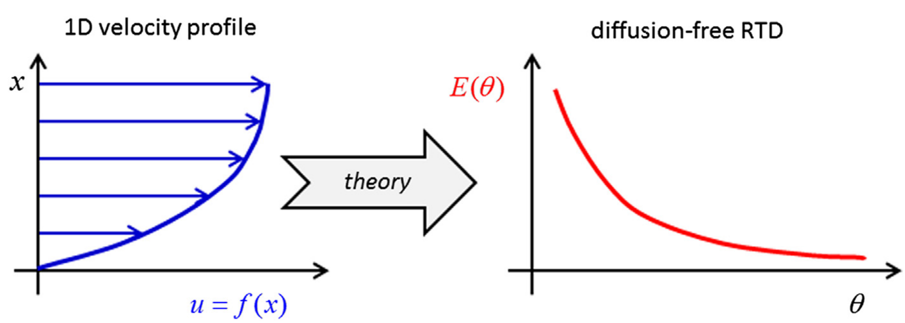
HIGHLIGHTS

- The diffusion-free RTD of fully developed laminar flow is studied analytically.
- A general relation for computing the RTD of a given 1D velocity profile is derived.
- The relation applies to straight planar and axisymmetric channels.
- The theory is unifying and comprises all pure convection RTDs derived so far.
- The theory is used to derive the RTD of general plane Couette–Poiseuille flow.

GRAPHICAL ABSTRACT



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ABSTRACT

In literature, the diffusion-free residence time distribution (RTD) of laminar flows – the so-called convection model – has been determined for various velocity profiles mostly on a case-by-case basis. In this analytical paper, we derive general mathematical relations which allow computing the diffusion-free differential and cumulative RTD in straight planar, circular and concentric annular channels for arbitrary monotonic and piece-wise monotonic one-dimensional velocity profiles. The theory is used to determine the RTD of plane Couette–Poiseuille flow with non-monotonic velocity profile, and the optimal value of the volumetric flow rate where the RTD becomes most narrow. It is shown that any velocity profile that depends in a sub-layer linearly on the distance from a stationary or moving no-slip wall has a differential RTD which follows a -3 power law as the residence time approaches its maximum. The variance of the RTD is directly associated with the asymptotic behavior of the RTD and can be finite or infinite.

1. Introduction

Reactors with laminar flow are frequently used for processing viscous fluids such as liquid food products (Torres and Oliveira, 1998) or polymers (Gogos et al., 1987), and in micro-process engineering (Kockmann, 2006). Recent progress in microfluidic technology has turned attention to the residence time distribution (RTD) in micro-devices (Cantu-Perez et al., 2010; Lohse et al., 2008; Vikhansky, 2011; Wibel et al., 2013). The velocity profile of laminar flow causes fluid elements to spent different times within the reactor and gives rise to a wide RTD. In the pure convection regime, molecular diffusion is negligible and each fluid element follows its streamline with no intermixing with neighboring elements. This corresponds to 'macrofluid behavior' so that the average conversion can be predicted by the total segregation model (Danckwerts, 1958; Fogler, 1986; Zwietering, 1959).

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This model assumes that fluid elements having the same age (residence time) 'travel together' in the reactor and do not mix with elements of different ages until they exit the reactor (Madeira et al., 2006). Levenspiel (1999) provides a map which recommends using the pure convection model for situations, where the Bodenstein number is sufficiently large while the ratio between channel length and diameter is sufficiently low.

In this paper we are interested in the RTD of this pure convective regime for general laminar flows. Throughout the paper we assume that the ratio of the length to the hydraulic diameter of the channel is sufficiently long so that entrance effects can be ignored; for a discussion of entrance effects on the RTD we refer to Ham et al. (2011). Under these conditions, the cumulative RTD, $F(\theta)$, and the non-dimensional differential RTD, $E_\theta(\theta) = \mathrm{d}F(\theta)/\mathrm{d}\theta$, depend on the fully developed velocity profile only and are fully deterministic. Closed analytical forms of the diffusion-free RTD in laminar flows are known only for very few channel shapes and certain Newtonian, non-Newtonian or generalized velocity profiles, see Table 1. Examples are axisymmetric flows in a circular pipe (Bosworth, 1948; Danckwerts, 1953; Delaplace et al., 2008; Hsu and Wei, 2005; Osborne, 1975; Pegoraro et al., 2012; Sawinsky and Balint, 1984; Sawinsky and Deak, 1995; Wein and Ulbrecht, 1972; Zakharov, 2006), in a planar channel (Asbjornsen, 1961; Levenspiel et al., 1970; Sawinsky and Simandi, 1983; Zakharov, 2006) and in a concentric annulus (Nigam and Vasudeva, 1976). In all these cases the velocity profile is one-dimensional as it depends on one coordinate only. For fully developed two-dimensional laminar flows, the diffusion-free RTD has been obtained approximately for helically coiled tubes (Nauman, 1977) and rectangular channels (Wörner, 2010), and analytically exact for channels with moon-shaped cross-section (Erdogan and Wörner, 2013).

The mathematical procedure to compute the RTD from the given laminar velocity profile is always similar. However, it has been applied mostly in a case-by-case manner. While Wein and Ulbrecht (1972) and Nauman (1974) derived a general expression for computing the RTD from a given monotonic velocity profile in a circular pipe flow, their relations are almost unknown to the community and have obviously not been used by other authors so far. In this paper we extend these studies and derive, for the first time, an explicit relation which allows computing the RTD from arbitrary one-dimensional monotonic or non-monotonic (but piecewise monotonic) laminar velocity profiles in planar, circular and annular channels. This relation includes all previous results from literature as special cases.

In Section 2 of this paper we give some basic definitions and provide a general mathematical expression for computing the convection model RTD from a given laminar velocity profile. In Section 3 we apply the new theory and determine for various velocity profiles the corresponding RTD. We recover some known results from literature but also present new results not obtained so far. In Section 4 we discuss the implications of the theory on the asymptotic behavior and the variance of the RTD. The paper closes by a short summary and conclusions in Section 5.

2. General pure convection RTD theory

2.1. Basic definitions

We consider a straight flow domain with length L, constant cross sectional area A, volume $V = LA$, constant total volumetric flow rate Q_{total}, mean velocity $U_m = Q_{total}/A$, and mean (hydrodynamic) residence time $t_m = V/Q_{total} = L/U_m$. We assume that the flow in this domain is steady, unidirectional (axial) and obeys a given fully developed laminar one-dimensional velocity profile.

In the sequel, we express all results in terms of the non-dimensional time $\theta = t/t_m$. The non-dimensional residence time of the fastest fluid elements is commonly denoted as first-appearance time or break-through time; it is given by $\theta_F = U_m/U_{max}$, where U_{max} is the maximum velocity of the profile. For $\theta < \theta_F$ it is $E_\theta = F = 0$; the equations given for E_θ and F in the remainder of the paper refer to $\theta \geq \theta_F$ only.

For any RTD, the zero and first moment are unity, i.e.

$$\mu_0 = \int_0^\infty E_\theta \mathrm{d}\theta = 1, \quad \mu_1 = \int_0^\infty \theta E_\theta \mathrm{d}\theta = 1 \tag{1}$$

The definition of the variance is

$$\sigma_\theta^2 = \int_0^\infty (\theta-1)^2 E_\theta \mathrm{d}\theta$$
$$= -1 + 2\int_0^\infty \theta(1-F)\mathrm{d}\theta = -1 + \theta_F^2 + 2\int_{\theta_F}^\infty \theta(1-F)\mathrm{d}\theta \tag{2}$$

Table 1
Overview on pure convection RTDs for one-dimensional velocity profiles derived in previous literature for different fluids and flow type/driving-force (C=Couette flow, E=electro-osmotic driven, F=falling film flow, G=generalized velocity profile, P=pressure-driven flow). RTDs which are first derived in the present paper are also included.

Geometry	Fluid/velocity profile	Flow type	Reference
Planar	Newtonian	C	Levenspiel et al. (1970)
	"	F	Asbjornsen (1961)
	"	P	Levenspiel (1989)
	"	$C+P$	Levenspiel et al. (1970) only monotonic case; present paper: non-monotonic case
	Bingham	F	Zakharov (2006)
	Ostwald-de Waele	C	Foraboschi and Vaccari (1965)
	"	F	Zakharov (2006)
	Prandtl–Eyring	F	Sawinsky and Simandi (1983)
	Generalized root law	G	Present paper
Circular pipe	Newtonian	P	Bosworth (1948), Danckwerts (1953)
	"	$P+E$	Hsu and Wei (2005)
	Bingham	P	Wein and Ulbrecht (1972)
	Casson	P	Sawinsky and Balint (1984)
	Herschel–Bulkley	P	Wein and Ulbrecht (1972)
	Ostwald-de Waele	P	Foraboschi and Vaccari (1965), Novosad and Ulbrecht (1966)
	Prandtl–Eyring	P	Wein and Ulbrecht (1972)
	Rabinowitsch	P	Wein and Ulbrecht (1972)
	Generalized root law	G	Bosworth (1949)
	Generalized exponential and sinusoidal law	G	Pegoraro et al. (2012)
Concentric annulus	Newtonian	P	Nigam and Vasudeva (1976) in implicit form; present paper: explicit form
	Ostwald-de Waele	P	Lin (1980) approximate; Pechoc (1983) numerical