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Geometric output tracking of nonlinear distributed parameter systems via adaptive model reduction

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HIGHLIGHTS

- A geometric dynamic observer is introduced for distributed parameter systems.
- The output feedback scheme includes observer and globally linearizing controller.
- The proposed controller is designed to solve the output tracking problem for DPS.
- The closed-loop stability of the DPS is proved under relaxed conditions.

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ABSTRACT

We focus on the output tracking problem of distributed parameter systems (DPSs) which can be described by a set of nonlinear dissipative partial differential equations (PDEs). The infinite-dimensional modal representation of such systems in appropriate subspaces can be decomposed to finite-dimensional slow and probably unstable, and infinite-dimensional fast and stable subsystems. Taking advantage of this decomposition, adaptive model reduction techniques and specifically adaptive proper orthogonal decomposition (APOD) can be used for the recursive construction of locally accurate low dimensional reduced order models (ROMs). The proposed geometric APOD-based control structure is the combination of a nonlinear Luenberger-like geometric dynamic observer and a globally linearizing controller (GLC) designed for tracking the desired output. The proposed geometric control approach is successfully illustrated on the output tracking of target thermal dynamics for a catalytic reactor. Specifically, the geometric output tracking strategy is used to reduce the hot spot temperature and manage the thermal energy distribution through reactor length during process evolution with limited number of actuators and sensors.

1. Introduction

Recent interest in monitoring, optimization and control of distributed parameter systems (DPSs) has significantly increased in industrial chemical processes. This interest has specifically grown in the processes that involve the coupling of chemical reactions and diffusion/convection/dispersion mechanisms such as bioreactors, catalytic reactors, lithographic operations, crystallization and polymerization processes (Armaou and Christofides, 1999; Christofides and Daoutidis, 1997; Garcia et al., 2012; Smyshlyaev and Krstic, 2005). It is imperative to tightly control these processes so that there are zero product quality excursions, even when the process objectives dynamically change during operation, a usual occurrence in chemical industries. The task of regulation and output tracking of such systems is challenging due to spatial dependency of the system dynamics, and requires a particular set of modeling and control tools to deal with the spatio-temporal development of the objectives. Direct approaches based on the infinite dimensional system theory are applied to control of standard linear and bilinear DPSs (Balogh and Krstic, 2002; Byrnes et al., 1994; Curtain, 1986; Curtain and Glover, 1986; Gauthier and Xu, 1991; Krstic, 2009; Lasiecka, 1995).

Spatially distributed processes with reaction and significant diffusion terms can be mathematically modeled by a set of nonlinear dissipative partial differential equations (PDEs) (Christofides, 2000). Consequently, the infinite-dimensional representation of such systems in appropriate Sobolev subspaces can in principle be decomposed to a finite-dimensional slow and possibly unstable, and an infinite-dimensional fast and stable, subsystems. Taking advantage of this decomposition, such PDE systems can be approximated by a finite-number of ordinary differential equations (ODEs). In detail, the solution of the PDE system can be presented as an

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infinite-sum of spatial basis functions times time-varying coefficients called modes. Then the infinite-dimensional ODE system for the modes can be derived using variants of weighted residual method. The finite-dimensional reduced order model (ROM) can then be derived by considering a proper number of ODEs corresponding to the basis functions which capture the dominant dynamics of the original PDE system. This standard strategy, named model order reduction (MOR), has been frequently applied to address control, monitoring and optimization problems of dissipative PDEs (Balas, 1979; Demetriou and Kazantzis, 2004).

On the basis of the above, when the set of basis functions is computed, the PDE systems can be discretized and the ROMs can be constructed. However the applicability of analytical model reduction methods to industrial processes is limited due to complex nonlinear spatial dynamics and irregular domains. Statistical techniques like proper orthogonal decomposition (POD) are usually used to bypass this limitation and construct empirical basis functions (Armaou and Christofides, 1999, 2001; Pourkargar and Armaou, 2013b; Christofides and Armaou, 2000; Pitchaiah and Armaou, 2010; Sirovich, 1987). POD has been applied extensively in model reduction, optimization and control of DPSs (Armaou and Christofides, 1999, 2001; Christofides and Armaou, 2000; Izadi and Dubljevic, 2013). It assumes the a priori availability of a sufficiently large ensemble of PDE solution data in which the most prevalent spatial modes are excited. To circumvent this requirement an efficient recursive computation algorithm, known as adaptive proper orthogonal decomposition (APOD), can be used as additional data from the process becomes available. APOD is comprehensively described in Pitchaiah and Armaou (2010) and Varshney et al. (2009); it is based on algebraic manipulations leading to a three-fold increase in computational speed compared to brute-force streaming methods and similar optimization based techniques allowing for its on-line implementation. A modification to APOD based on information theory concepts was introduced for regulation of DPSs with fast transients (Pourkargar and Armaou, 2013b). The novelty of the approach lies in modifying the data ensemble revision steps within APOD to enlarge the ROM region of attraction. The requirements on continuous measurement sensors were then reduced using APOD-based dynamic observers (Pourkargar and Armaou, 2013a, in press a). In addition, a criterion is characterized to minimize the communication bandwidth from the distributed sensors to the APOD-based control structure considering closed-loop stability to identify how infrequent the ROM revisions can be (Pourkargar and Armaou, in press b).

Generally, the existence of nonlinearities in chemical process systems is the rule rather than the exception. Chemical reactions and complex fluid dynamics are prevalent sources of nonlinearities in such systems. Then the control design problem is challenging due to the complexity of the systems' dynamics and their governing models (Kravaris and Kantor, 1990a). Recently, the use of geometric methods in nonlinear process control and observer design has significantly increased by reason of their effectiveness in considering complex nonlinear dynamic of the systems (see Alvarez, 2000; Alvarez and Lopez, 1999; Kravaris and Kantor, 1990a, 1990b; Tronci et al., 2005 and references therein). This approach can be directly applied to many nonlinear process systems including reduced models of nonlinear DPSs where nonlinearity plays an important role in their dynamics.

In this paper, an APOD-based geometric output feedback control structure is synthesized for output tracking of nonlinear DPSs based on continuous point measurements available from limited number of sensors. The control structure is a combination of a nonlinear Luenberger-like geometric dynamic observer and a globally linearizing controller (GLC). The specific structure is employed to compensate model uncertainty due to model reduction procedure. Section 2 introduces a few mathematical preliminaries used through the paper. A short review on adaptive model reduction is presented in Section 3. It includes the off-line and on-line computation of empirical basis functions and ROM construction. The APOD-based geometric dynamic observer and controller designs are described in Sections 4 and 5, respectively. Finally, in Section 6 the proposed control structure is successfully illustrated on a catalytic reactor. The controller considers the thermal dynamics of the reactor, reduces the hot spot temperature and manages the thermal energy distribution across the reactor length during process operation.

2. Preliminaries

2.1. Class of nonlinear dissipative PDE systems

A class of nonlinear dissipative, input-affine PDE systems is considered with a state space representation of the following form:

$$\begin{aligned}
\frac{\partial}{\partial t}\overline{x}(z,t) &= \mathcal{A}(z)\overline{x}(z,t)+\mathcal{F}(z,\overline{x})+b(z)u(t),\\
y_c(t) &= \int_{\Omega} c(z)\overline{x}(z,t)\,dz,\\
y_m(t) &= \int_{\Omega} s(z)\overline{x}(z,t)\,dz,\\
y_r(z,k) &= \int_0^t \delta(t-t_k)\overline{x}(z,t)\,dt,
\end{aligned} \tag{1}$$

subject to boundary conditions

$$q\left(\overline{x},\frac{\partial\overline{x}}{\partial z},\ldots,\frac{\partial^{n_0-1}\overline{x}}{\partial z^{n_0-1}}\right)=0 \quad \text{on } \partial\Omega, \tag{2}$$

and initial condition

$$\overline{x}(z,0)=\overline{x}_0(z), \tag{3}$$

where $\overline{x}(z,t)\in\mathbb{R}$ denotes the vector of state variables and $u(t)\in\mathbb{R}^l$ is the vector of manipulated inputs. t is the time, $z\in\Omega\subset\mathbb{R}^3$ denotes the spatial coordinate and Ω is the process domain with boundary, $\partial\Omega$. $\mathcal{A}(z)$ and $\mathcal{F}(z,\overline{x})$ are linear and bounded Lipschitz nonlinear parts of spatial differential operator of order n_0, respectively. $b^T(z)\in\mathbb{R}^l$ is a smooth vector function of z that describes how the control action is distributed in the spatial domain, e.g. point actuation is defined using standard Dirac delta. $q(\cdot)$ is a sufficiently smooth nonlinear vector function, $\partial^i\overline{x}/\partial z^i|_{\partial\Omega}$ for $i=1,\ldots,n_0-1$, denotes the spatial derivatives in the direction perpendicular to the boundary and $\overline{x}_0(z)$ is a smooth vector function of z. $y_c\in\mathbb{R}^{\upsilon}$ is the vector of controlled outputs where υ is the number of desired outputs. $c(z)$ is a known vector function of z which is determined by the desired performance specifications in the process domain, Ω. We assume that two types of measurement sensors are available during process evolution: periodic distributed snapshot measurements, $y_r(z,k)\in\mathbb{R}$, and continuous measurements, $y_m\in\mathbb{R}^w$, where w is the number of continuous sensors and k is a discrete variable that indicates the sample time counter to taking the snapshots. Note that y_r indicates measured spatial profiles while y_m is a vector variable. $s(z)$ is the sensor shape functions corresponding to y_m and t_k is the time instance for snapshot measurement. In this paper, the results are presented for $\overline{x}\in\mathbb{R}$, however, it is straightforward to extend them for $\overline{x}\in\mathbb{R}^n$, by treating each state independently (Sirovich, 1987). Once each state has been reduced, the interactions between distributed system states can be easily captured through the inner products of the modal expansions.

2.2. Infinite-dimensional representation in Sobolev subspace

To address the control and observation problem we represent the PDE system of (1)–(3) as an infinite-dimensional system in a