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On ideal and optimum cascades of gas centrifuges with variable overall separation factors

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HIGHLIGHTS

- We compare the model optimum and ideal cascades of gas centrifuges.
- The overall separation factors of gas centrifuges vary over cascade stages.
- We examine gas centrifuges with various separation performances.
- The reduction in a total flow in the optimum cascades is demonstrated.

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ABSTRACT

It is demonstrated that the total flow in the optimum cascade composed of three different types of gas centrifuges with varied separative performance levels and overall separation factors varying over the cascade stages is less than that in the corresponding ideal (non-mixing) cascade. The data obtained make it possible to use the parameters of the optimum cascade to find the operational mode for the cascade stages that provides the minimum total flow in a separation cascade.

1. Introduction

In the classical theory of multistage separation installations (cascades), it has been shown that in an ideal (non-mixing) cascade, the total interstage flow (or the total number of separation elements, which is equivalent) is minimal (Cohen, 1952). Until recently, this statement was considered inviolable. At the end of 20th and the beginning of 21st centuries, when the concept of the optimum in the total flow cascade for binary isotope mixtures was invented, it became clear that this assertion is not always valid (Palkin, 1998; Borisevich et al., 2003; Sulaberidze et al., 2003; Sulaberidze et al., 2008b; Song and Zeng, 2006; Norouzi et al., 2011). It was demonstrated that the total flow in the optimum cascade composed of gas centrifuges with overall separation factors noticeably higher than unity is less than that in an ideal cascade, although the former permits mixing concentrations in the confluent points of the cascade. The advantage of the optimum cascade in comparison with an ideal one is explained by better usage of the separative performance of the gas centrifuges with different working regimes in the various parts of the cascade.

The evident advantages of the optimum cascade for binary isotope mixtures led to swift generalization of this concept for the case of multi-isotope mixtures (Sulaberidze et al., 2008a; Song et al., 2010; Palkin et al., 2002). Note that in contrast to the classical theory of isotope separation in cascades, in which a non-mixing (ideal) cascade cannot be built for multi-isotope mixtures, the concept of the optimum cascade is applicable to any type of cascade without limitations to mixtures with any specific number of components.

In the classical theory of separation cascades, as well as in the newest concept of the optimum one, the most published works on cascade performance optimization are devoted to the case of the constant overall stage-separation factor, although it actually varies from stage to stage. In a common practical case of gas centrifugation, the overall separation factor in fact depends on the composition of the separating mixture, the cut, the feed-flow rate to a GC,

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and also the holdup, as is known from practice and theory (cf. Soubbaramayer, 1979; Wood, 2006). Note that the theory of an ideal cascade was generalized for varying the overall separation factors long ago (Olander, 1976). Treatment of the separation factor as a constant can ease very much cascade performance analysis, but might also lead to a significant deviation of results from reality. Because the separation factor is large in the separation by gas centrifugation, it is necessary to compare the separation performances of cascades with and without this treatment and see the differences.

The purpose of this paper is to define whether the total flow (number of gas centrifuges) in the optimum cascade for a binary isotope mixture with varied separation factors is less than in the ideal cascade, and if yes, to what extent. Note that, for constant overall separation factors, this gain is approximately a few percent or less (Sulaberidze et al., 2008a). Therefore, is it of great practical importance? Here, we should note the following. An industrial separation cascade for uranium enrichment is usually very large in scale and consists of tens of thousands of gas centrifuges (GCs). Therefore, even a one-percent difference in the total flow suggests a considerable economic benefit. Thus, it is worthwhile to carefully consider the optimization of a cascade at the phase of its design, based on the particular separation characteristics of a GC in use.

2. The governing equations

The object of the research is a counter-current symmetric cascade for the separation of binary isotope mixtures. The external parameters of a cascade to be calculated are the feed F, product P and waste W flows; the concentrations of a target component in each of these flows, C_F, C_p, and C_W, respectively; the total number of stages in a cascade n; and the number of the stage in which the feed flow enters a cascade f (see Fig. 1). In the problem under investigation, it is assumed that the working regimes of GCs at the cascade stages are different.

The internal parameters of a cascade are the stage characteristics. These are the feed, product and waste flows L_s L'_s, L''_s, and the C_s, C'_s, C''_s concentrations of the U-235 isotope in them, respectively, as well as a stage cut θ_s and the overall separation factor q_s. They can be found by the recurring formulas (Palkin, 1997; Sulaberidze et al., 2003):

$$C'_s = \frac{q_s C''_s}{1+(q_s-1)C''_s} \tag{1}$$

$$C''_1 = C_W; \quad C''_s = C'_{s-1}\left(1-\frac{T'_s}{L''_s}\right)+\frac{T'_{us}}{L''_s} \tag{2}$$

$$q_s = q_s(g_s;\theta_s), \quad g_s = L_s/N_s, \quad L_s = L''_{s+1}+L'_{s-1}, \tag{3}$$

where N_s is the number of GCs in the sth cascade stage, g_s is a feed-flow rate to a single GC, and T'_s and T'_{us} are the transit flow of a separating mixture as a whole and that of a target component, respectively. As a result, we have the following chain of equalities: $T'_s = W$, $T'_{us} = WC_W$ for $s<f$ and $T'_s = -P$, $T'_{us} = -PC_P$ for $f<s<n$. Next, the concentrations in the enriched and depleted fractions at

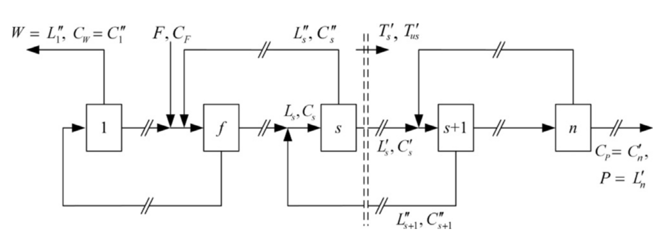

Fig. 1. Schematic drawing of a separation cascade.

all stages are determined. Determining the concentration C_n in the last stage completes the calculation procedure. Finally, it is necessary to check the boundary condition $C'_n = C_p$. If it is valid, it means that all the parameters have been defined correctly and one may avoid searching other parameters. Otherwise, one of the defined parameters should be changed, and the procedure above must be repeated.

In mathematical terms, the search for the most efficient cascade is equivalent to searching for the minimum of the function $\Psi = \Sigma_s L_s$ over the set of possible values for n, f, L_s $(s=1,2,\ldots,n)$ that satisfy the following conditions:

$$C'_n = \Phi(L''_2, L''_3, \ldots, L''_n) = C_P, \tag{4}$$

where the function Φ represents the procedure to calculate the component concentrations over the cascade stages. The function Φ is written in an explicit form by substituting the parameters q_n and C''_n expressed in terms of L''_n and C'_{n-1} from Formula (2) into Formula (1) with $s=n$. Then, we write C'_{n-1} and the concentrations of all the preceding stages analogously:

$$\begin{cases} C'_n = C'_n(L''_n, C'_{n-1}) \\ C'_{n-1} = C'_{n-1}(L''_{n-1}, C'_{n-2}) \\ \cdots\cdots\cdots\cdots C'_1 = C'_1(L''_1) = const. \end{cases}$$

Supposing that the flow L''_2 is determined in terms of Φ from (4), i.e., it is an implicit function of $L''_3, \ldots, L''_n$, the necessary conditions for the minimum of ψ are written as:

$$\frac{\partial\psi}{\partial L''_s} = 0 \Rightarrow \frac{\partial L''_2}{\partial L''_s}+1 = 0, s = \overline{3,n}. \tag{5}$$

Because the equation as follows is valid:

$$\frac{\partial L''_2}{\partial L''_s} = -\frac{\partial\Phi}{\partial L''_s}\Big/\frac{\partial\Phi}{\partial L''_2},$$

taking into account (5), one can obtain

$$\frac{\partial\Phi}{\partial L''_s} = \frac{\partial\Phi}{\partial L''_2}, s = \overline{3,n}.$$

Using the obvious interstage balance of flows $L''_s - L'_{s-1} = T'_s$, the dependences of the overall separation factors are as follows:

$$\begin{cases} q_1 = q_1(L''_2); q_n = q_n(L''_n) \\ q_s = q_s(L''_s, L''_{s+1}), s = \overline{2, n-1} \end{cases}.$$

Then, the function Φ is presented in the explicit form as a function of $L''_2, L''_3, \ldots, L''_n$.

The described calculation method makes it possible to calculate the stage parameters from the waste to the product end of a cascade satisfying the condition of its optimality automatically.

To make the system of equations closed, it is necessary to know in Formulas (3) the dependence of q_s on a feed-flow rate g_s and a cut θ_s, for a single GC that differs from stage to stage.

3. Parameters of a single GC

3.1. Gas centrifuge no. 1

The function of an overall separation factor q on a cut θ and a feed-flow rate g can be found from the results of calculation or experiments. For this purpose, we used the results of the calculation of the one-dimensional dependences $q(g)$ for various cuts from a paper (Migliorini et al., 2013) for the hypothetical Iguassu GC (Schwab et al., 1996) that will be further denoted as GC1. These dependences are approximated by the surface with the help of the least-squares method. The graphical illustration of the data used and the approximation surface is given in Fig. 2. This approximation resulted in the