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Numerical study of mixed convection within porous square cavities using Bejan's heatlines: Effects of thermal aspect ratio and thermal boundary conditions

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ABSTRACT

The present numerical study deals with mixed convection flows within square enclosures filled with porous media. The influence of various thermal boundary conditions on bottom and side walls based on thermal aspect ratio (A) is investigated for a wide range of parameters ($1 \leqslant Re \leqslant 100$, $0.015 \leqslant Pr \leqslant 7.2$, $10^{-5} \leqslant Da \leqslant 10^{-3}$ and $10^3 \leqslant Gr \leqslant 10^5$). A penalty finite element method with bi-quadratic elements has been used to investigate the results in terms of streamlines, isotherms and heatlines and average Nusselt numbers. Lid driven effect is dominant at low Darcy number ($Da = 10^{-5}$), whereas buoyancy driven effect is dominant at high Darcy numbers ($Da = 10^{-4}$ and $Da = 10^{-3}$) for $Re = 1$. Asymmetric pattern is observed in isotherms and heatlines for $Re = 100$. It is found that thermal gradient is high at the center of the bottom wall for $A = 0.1$ due to large dense heatlines at that zone and that is low for $A = 0.9$ irrespective of Re, Pr and Gr. Overall heat transfer rates are higher for $A = 0.1$ compared to other thermal aspect ratios ($A = 0.5$, $A = 0.9$) irrespective of Darcy number, Prandtl number and Reynolds number.

1. Introduction

The investigations on transport phenomena through saturated porous media are substantially increased in recent years due to its numerous practical applications encountered in various fields of engineering and natural science. Among these investigations, natural and forced convection studies occupied major position. The combined effect of natural and forced convection is commonly referred as mixed convection. Mixed convection flow within porous cavities may occur in many technical applications such as geothermal engineering [1], energy recovery [2], packed bed reactors [3], electronic cooling [4], chemical reactions [5] etc. The governing non-dimensional parameters for mixed convection in a cavity filled with fluid saturated porous medium are Darcy number (Da), Grashof number (Gr), Reynolds number (Re) and Prandtl number (Pr). Note that, Gr and Re represent the strength of the natural and forced convection flow effects, respectively. In addition, another dimensionless parameter, Richardson number (Ri), may be defined as $Ri = Gr/Re^n$ characterizes the mixed convection flow. A comprehensive review on the fundamentals of the convective flow in porous media can also be found in the literatures [6–14].

A few earlier investigations have been conducted on mixed convection flow within lid driven porous cavities. Khanafer and Chamkha [15] reported mixed convection flow in a lid-driven enclosure filled with a Darcian fluid saturated porous medium in the presence of internal heat generation. Oztop [16] analyzed heat transfer and fluid flow in a partially heated porous lid driven cavity with isothermal moving top wall. Khanafer and Vafai [17] analyzed mixed convection heat and mass transport in a lid-driven square cavity filled with a non-Darcian fluid-saturated porous medium. The two vertical walls of the enclosure are insulated, while the horizontal walls are kept at constant but different temperatures and concentrations with the top surface moving at a constant speed. Kandaswamy et al. [18] investigated mixed convection heat transfer in a lid-driven square cavity filled with a fluid-saturated porous medium. The left and right walls of the cavity are insulated, whereas horizontal walls are kept at constant but different temperatures. The top horizontal wall is moving in its own plane at a constant speed, while all other walls are fixed. Vishnuvardhanarao and Das [19] investigated mixed convection heat transfer in a lid driven square cavity filled with a fluid-saturated porous medium where left and right walls are moving upwards with same velocity. The right wall is hot and left wall is maintained at constant cold temperature. The top and bottom walls are fixed and are thermally insulated. However, most of the above numerical investigations have been analyzed based on streamlines and isotherms and the detailed analysis of heat flow was not well understood. Streamlines

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Nomenclature

Da	Darcy number
A	thermal aspect ratio
g	acceleration due to gravity, m s^{-2}
k	thermal conductivity, W m^{-1} K^{-1}
H	heatfunction
L	length of the square cavity, m
K	permeability, m^2
Nu	local Nusselt number
p	pressure, Pa
P	dimensionless pressure
Pr	Prandtl number
Ra	Rayleigh number
T	temperature, K
T_h	temperature at the bottom edges of the side walls, K
T_c	temperature at the top edges of the side walls, K
T_H	temperature at the center of the bottom wall, K
u	x component of velocity
U	x component of dimensionless velocity
v	y component of velocity
V	y component of dimensionless velocity
X	dimensionless distance along x coordinate
Y	dimensionless distance along y coordinate
Greek symbols	
α	thermal diffusivity, m^2 s^{-1}
β	volume expansion coefficient, K^{-1}
γ	penalty parameter
θ	dimensionless temperature
ν	kinematic viscosity, m^2 s^{-1}
ρ	density, kg m^{-3}
Φ	basis functions
ψ	streamfunction
ξ	horizontal coordinate in a unit square
η	vertical coordinate in a unit square
Subscripts	
b	bottom wall
l	left wall
r	right wall

adequately depict fluid flow whereas isotherms indicate only temperature distribution which may not be useful as heat flow would no more be in normal direction to isotherms for a convection dominant regime.

The heatline technique is the best way to visualize the heat transfer occurring in a two-dimensional convective transport process. The technique was first proposed by Kimura and Bejan [20] and Bejan [21] which can be used to illustrate the path of heat flow, its magnitude and zones of high heat transfer. Significant number of studies were carried out to visualize the heat flow for natural convection in various applications [22–26]. Zhao et al. [22] studied double-diffusive convective flow of a binary mixture in a porous enclosure subject to localized heating and salting from one side. Varol et al. [23] used heatlines for natural convection heat transfer in porous triangular enclosures with three different boundary conditions (cases 1–3). Case 1 represents isothermal vertical and inclined walls whereas case 2 corresponds to adiabatic vertical wall and isothermal inclined wall while case 3 involves isothermal vertical wall and adiabatic inclined wall. In all three cases bottom wall is non-isothermally heated. Hakyemez et al. [24] analyzed heatlines to investigate the influence on a heat barrier located in the ceiling wall of an enclosure, on conjugate conduction/natural-convection heat transfer. Deng [25] investigated laminar natural convection in a two-dimensional square cavity due to two and three discrete heat source-sink pairs on the vertical side walls. Dalal and Das [26] carried out natural convection inside a two-dimensional cavity with a wavy right vertical wall whereas the bottom wall is heated by a spatially varying temperature and other three walls are kept at constant lower temperature. Recently, Kaluri and Basak [27] analyzed the heat flow during natural convection within discretely heated porous square cavities using heatlines concept. They studied various cases depending on the location of discrete heat sources on the walls of the cavity. However, a comprehensive analysis of influence of thermal aspect ratio during mixed convection in porous media based on heatlines is yet to appear in literature.

The current study deals with mixed convection within square cavity filled with porous media for various thermal boundary conditions based on thermal aspect ratio on bottom and side walls where the top wall is adiabatic. An important non-dimensional parameter thermal aspect ratio (A) is considered for thermal boundary conditions. By varying thermal aspect ratio (A) from 0 to 1, various thermal boundary conditions are imposed on the bottom and side walls, such as $A = 0$ corresponds to non-uniformly heated bottom wall and cold side walls whereas $A = 1$ corresponds to uniformly heated bottom wall and linearly heated side walls.

In the present study, numerical investigations on heating characteristics within square cavities have been carried out based on coupled partial differential equations of momentum and energy which are solved using Galerkin finite element method with penalty parameter to obtain the numerical simulations in terms of streamfunctions, isotherms and heatfunctions. The streamfunctions and heatfunctions of Poisson equation are also solved using Galerkin finite element method. The jump discontinuity in Dirichlet type of wall boundary conditions for temperature at the corner points is tackled by implementation of exact boundary conditions at those singular points. The heating and thermal mixing have been studied for various lid velocities in terms of Re ranging from 1 to 100. Also, a number of model fluid for almost all industrial application ranges ($0.015 \leqslant Pr \leqslant 7.2$) have been considered for current analysis.

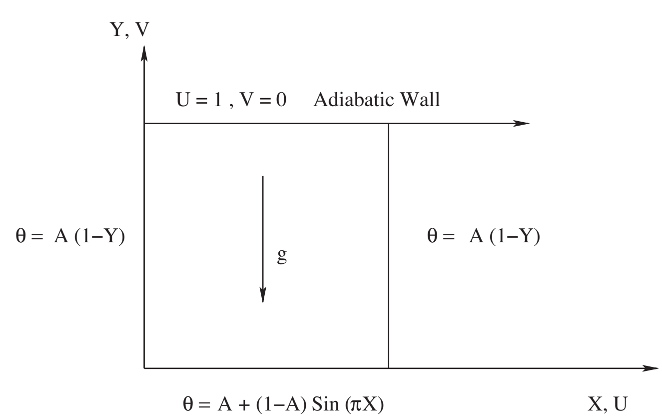

Fig. 1. Schematic diagram of the physical system.