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What is the most suitable fixed grid solidification method for handling time-varying inverse Stefan problems in high temperature industrial furnaces?

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ABSTRACT

This paper presents a simple inverse heat transfer method for predicting the time-varying bank thickness of the phase change protective layer inside a high temperature furnace. The direct problem is handled with the single phase method which is neglecting the latent heat. The inverse method rests on the Adjoint Problem and the Conjugate Gradient Method. It is shown that the iterative inverse procedure based on the single phase method predicts a virtual incident heat flux in the liquid zone that yields accurate time-varying bank predictions. Results also indicate that the benefits of the virtual iterative approach are twofold: the CPU time required for solving the inverse problem is reduced and the lagging effect inherent to the inversion is diminished for non isothermal phase change processes. For typical melting furnace operating conditions, it is shown that the virtual approach doubles the allowable diagnostic frequency for predicting the time-varying bank thickness.

1. Introduction

The purpose of this study is to predict the time-varying thickness of phase-change banks found on the inside surface of many high temperature industrial facilities such as electric arc furnaces [1] and aluminum reduction cells [2–4]. This fascinating solidification process, well described by [2], consists in the formation of a bank which is created as the molten material inside the furnace comes into contact with the cooled surface of the wall. Unfortunately, probing this protective bank with devices inserted into the bath of molten material is a very difficult task owing to the hostile conditions that prevail inside these furnaces. Therefore, investigations have been conducted to handle this problem by employing inverse methods [1,2,5–7].

This tracking inverse Stefan problem consists in inferring the bank's thickness based on temperature measurements made on the inside or at the outer surface of the furnace's sidewall. Due to the large thermal inertia of the phase change materials and of the furnace walls, the temperature measurements are plagued by long time delays with respect to the time-varying bank thickness [8]. As a result, one should choose a sliding time horizon sufficiently large in order to ensure the temporal stability of the inverse method [9]. In practice, the few investigations that have been reported in the open literature and that have addressed this inverse Stefan problem generally assume that the temporal stability condition is satisfied [2,5,10].

Most of the published studies dealing with the lagging effect have focused on the effect of the sensors locations [10,11]. Indeed, it is a well known fact that the closer the sensor to the moving boundary location, the better the predictions [12]. For practical reasons however, it is desirable to place the sensors on the outer surface of the sidewall. Hence, the lagging effect is taken into account by increasing the sliding time horizon [8]. The minimum sliding time horizon becomes however rapidly prohibitive in an industrial context. Indeed, the required sliding time horizon usually exceeds the timescale of the phenomena occurring inside the furnace. In order to overcome such a difficulty, an overlapping procedure based on a sequential function estimation method has been proposed by Marois et al. [8]. The drawback of this technique is that it is CPU time consuming as the diagnostic frequency increases.

The present context is directed toward the identification algorithm that requires the resolution of the direct problem. Among the numerous existing approaches, fixed grid methods are efficient for the numerical modeling of the direct problem since the phase change process can be easily incorporated to existing heat transfer numerical methods [13]. Marois et al. have indicated however that the phase change process accounts for a large part of the undesirable lagging effect in melting furnaces [9]. As a result, though the general source-based enthalpy method promoted by Voller and Swaminathan is one of the most accurate fixed grid direct methods, it is not necessarily the most suitable fixed grid phase change method for inverse problem [14].

The objective of the study involves maximizing the diagnostic frequency that is the frequency at which values are estimated in the inverse problem. It is done through reducing the minimum

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Nomenclature

c	average heat capacity, J/kg K
c_p	heat capacity, J/kg K
ER_s	relative discrepancy
g	liquid fraction
h	convection coefficient, W/m^2 K
H	enthalpy, J
I	total number of sampling instants
k	thermal conductivity, W/m K
L	latent heat of fusion, J/kg
q	heat flux, W/m^2
r	number of sampling time steps
S	least square function
t	time, s
T	temperature, K
x, y	Cartesian coordinates, m
x_M	sensor location, m
Y	measured temperature, K
Δt	time step, s
Δt_m	sampling time step, s
Δx	mesh size, m
ΔX	dimension, m
ρ	Density, kg/m^3
σ	standard deviation of measurements, K
τ	time, s
τ_o	sliding time horizon, s
Ψ	predicted temperature, K
Ω	boundary condition

Subscripts

amb	ambient
diff	diffusion
f	final
in	incident
l	liquid
liq	liquidus
m	mushy
M	measured
min	minimum
o	initial value
PCM	phase change material
PCMl	liquid phase change material
PCMs	solid phase change material
s	solid
sol	solidus
tot	total
w	wall
x, *y*	Cartesian coordinates

Superscripts

–	estimated parameter
k	iteration number

sliding time horizon in such a way as to implement an inverse technique in an industrial environment. For that purpose, a new numerical inverse method based on the single phase method for solving the direct phase change problem detailed in [15] is proposed. The resulting inverse method relies on a concept of virtual area widely applied to nonlinear inverse heat transfer problems [16–18]. It rests on the Adjoint Problem and the Conjugate Gradient Method and it relies on non intrusive temperature measurements taken at the outer surface of the furnace wall.

In the next section, the problem under investigation is stated. Two different direct models used for handling the phase change process are then presented and compared. The formulation of the inverse problem is developed as well as the virtual concept. Both direct models that have been retained for the inverse procedure are tested and compared using a whole time scheme. Finally, it is shown that the single phase direct method used iteratively in the sequential inverse method is a promising technique for minimizing the inherent time delay found in high temperature furnaces.

2. Problem statement and assumptions

A schematic representation of the one-dimension phase-change problem is depicted in Fig. 1. The phase change material (PCM) is confined to a finite region $\varsigma = \varsigma_s \cup \varsigma_m \cup \varsigma_l$, including a solid zone ς_s, a mushy zone ς_m and a liquid zone ς_l. The top and the bottom boundaries, Ω_1 and Ω_2, are both adiabatic. A heat flux $q_{in}(t)$ mimicking the average heat dissipated through the sidewall of the furnace is imposed on the Ω_4 boundary over the time interval $t =]o, t_f[$. A convection heat transfer boundary condition is imposed at the outer surface of the furnace (Ω_3 boundary). The PCM is initially in steady-state with $q_{in}(t < t_o) = q_o$. The initial bank thickness is $x_{sol}(t_o) = x_o$.

A conduction-dominated model that takes into account the thermal convection effects in the bulk fluid region is adopted. Therefore, an enhanced thermal conductivity for the liquid zone mimics the effect of the flow circulation [1,2,19]. The following additional assumptions are made regarding the modeling of the phase change problem

1. The temperature gradients in the *x*-direction are much larger than that in the vertical *y*-direction and as a result a one-dimensional analysis is applied.
2. The phase change is non isothermal, i.e., melting and solidification occur over a temperature range.
3. The thermal contact resistance between the furnace wall and the bank is ignored.
4. The thermal properties of the bank are temperature independent.
5. The thermal conductivity and the heat capacity of the PCM vary linearly with the liquid fraction in the two-phase mushy zone [14].
6. The heat losses at the outer surface of the furnace wall accounts for both radiation and convection heat transfer.

Assumption 1 neglects the temperature gradients in the *y*-direction although it could be of importance near the top and the bottom boundaries, Ω_1 and Ω_2 [2]. Assumption 5 has been retained for its simplicity. The estimation of the thermal properties of the PCM in the mushy zone is beyond the scope of the present study. Furthermore, the authors recognize that a two-dimensional analysis would reveal conduction effects in the sidewall which are not captured by the present one-dimensional model [20]. Nevertheless, these 2D effects do not modify the conclusions of the present analysis concerning the virtual approach benefits.

3. The direct problem

3.1. The enthalpy method

The above phase-change problem is first handled with the general source-based enthalpy method promoted by Swaminathan