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Thermal rectification between two thermoelastic solids with a periodic array of rough zones at the interface

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ABSTRACT

An analysis of the effective thermal contact resistance between two semi-infinite solids in the presence of a periodic array of rough zones at the interface is carried out on the basis of a solution of the corresponding thermoelastic contact problem. The effect of the roughness is modeled by localized thermal contact resistances varying inversely with the contact pressure. The contact problem is reduced to a nonlinear singular integrodifferential equation, and an iterative procedure is proposed for its solving. The results demonstrate that the periodic array of rough zones between two semi-infinite solids exhibits thermal rectification. It is also found that the effective temperature jump and the effective thermal contact resistance are nonlinear functions of a far field heat flux.

1. Introduction

The effective thermal contact resistance of joints formed by rough surfaces is of interest in many fields including microelectronics, superconductors, nuclear engineering, bearings with lubrication, aerospace structures design, micro/nanoscale thermometry, and biomedical prosthetics.

When steady-state heat transfer takes place across the interface between two solids that are pressed together, the presence of surface roughness leads to the imperfect thermal contact. The imperfect thermal contact is characterized by the effective thermal contact resistance or the effective thermal contact conductance. The effective thermal contact resistance is defined as the ratio of the average temperature jump γ_{av} across the interface to the far field heat flux q^{∞} [1–8], i.e.

$$R_{eff} = \gamma_{av}/q^{\infty}. \quad (1)$$

The effective thermal contact conductance is the reciprocal of the effective thermal contact resistance.

The results of the experimental investigations showed that the thermal contact resistance varies with the contact pressure of surfaces of solids [3,8]. Several semi-empirical formulae which express the relationship between the thermal contact resistance and the contact pressure were proposed [3,6,9].

In most thermoelastic contact problems available in the literature, the pressure-dependent thermal contact resistance is considered to arise in the whole region of contact [10–14]. However, the local change of surface characteristics, in particular surface thermal resistance, can be caused due to the surface treatment by local melting, hardening or generating regular surface texture and discrete coatings that are often applied in the engineering practice [15–20]. At the same time, certain elements of contacting surfaces are in different conditions, subjected to oxidation, wear, destruction under the action of a medium, and contamination in different regions. Therefore, the investigation of the thermoelastic contact of solids in the presence of local areas with varying thermal resistance and determination of the effective thermal contact resistance on this ground are important from the practical point of view.

Dundurs and Panek [21] and Panek and Dundurs [22] solved the contact problem of heat conduction and the contact problem of thermoelasticity for two semi-infinite solids with wavy surfaces assuming that heat passes through the interface only where there is solid to solid contact. Later, Sadhal [23] obtained a long-time approximate solution for the corresponding transient problem of heat conduction. Comninou and Dundurs [24] studied the thermoelastic contact between solids with a periodic array of thermally insulated gaps at the interface. An analysis of thermal contact between solids with a periodic array of interfacial gaps filled with a conducting fluid was carried out by Das and Sadhal [25].

The thermoelastic contact of two half-spaces in the presence of one region with a thermal resistance which is not dependent from contact pressure, but which varies along this region, was previously

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studied by Shvets and Martynyak [26], Krishtafovich and Martynyak [27,28], Martynyak and Chumak [29]. Martynyak and Chumak [30] solved the thermoelastic contact problem when the localized roughness is present at the interface. The influence of the localized roughness on the heat transfer between solids was modeled by the pressure-dependent thermal resistance. The thermomechanical behavior of the bimaterial with the closed interfacial crack assuming thermal contact resistance between the crack faces due to surface films or roughness was analyzed by Martynyak et al. [31], Martynyak [32,33], Giannopoulos and Anifantis [34], Keppas and Anifantis [35], Giannopoulos et al. [36]. In the last five works, the thermal contact resistance was regarded as a function of contact pressure.

The goal of this study is to investigate the steady-state thermoelastic contact of two semi-infinite solids with a periodic array of rough zones at the interface and determine the effective thermal contact resistance for such a contacting couple.

2. Statement of the problem

The model for the present analysis and the orientation of the coordinate axes with respect to the two semi-infinite solids are shown in Fig. 1. The solids are pressed together by a nominal pressure p^{∞}, and a far field heat flux q^{∞} is imposed in the direction normal to the interface. The materials of the solids are assumed to be elastic, isotropic and dissimilar. The problem is posed in the framework of linear thermoelasticity, assuming plane strain conditions.

The surface of the upper solid S_1 is perfectly smooth, while the surface of the lower solid S_2 consists of a periodic array of rough zones $L=\bigcup_{m=-\infty}^{\infty}L_m,\ L_m=[-a+md,a+md]$, where the surface is rough, and a periodic array of smooth zones $L'=\bigcup_{m=-\infty}^{\infty}L'_m,\ L'_m=(a+md,-a+(m+1)d)$, where the surface is smooth. The lengths of the rough zones are denoted by $2a$. The length of a period is taken to be $d(d>2a)$.

The effect of the roughness is determined by considering the macroscopic thermal contact resistance $R(x)$ which occurs within each rough zone and varies inversely with the contact pressure $P(x)$, i.e.

$$R(x)=f(x)/P(x),\quad x\in L_m,\ m=0,\pm1,\pm2,\ldots \tag{2}$$

Here, $f(x)$ is a periodic function that describes the distribution of asperity heights in each rough zone.

The presence of the thermal contact resistances $R(x)$ leads to the temperature jumps $\gamma(x)$ across the interface within rough zones:

$$\gamma(x)=T^{-}(x,0)-T^{+}(x,0),\quad x\in L_m,\ m=0,\pm1,\pm2,\ldots \tag{3}$$

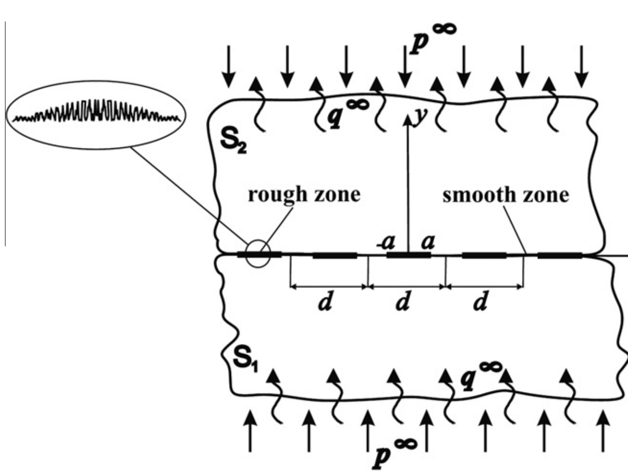

Fig. 1. Array of rough and smooth zones at the interface between semi-infinite solids S_1 and S_2.

Here, $T(x,y)$ is the temperature, the superscripts $^+$ and $^-$ denote the boundary values of the function on x-axis in the upper and lower solid, respectively.

It is assumed that radiation and the effect of an interstitial medium are insignificant, and there is no thermal contact resistance at the smooth part of the interface. For simplicity, the contact is taken as frictionless. It is also supposed that the global warping of the two solids is suppressed by applying linearly distributed along y-axis stresses σ_{xx}^{∞} far away [24].

While the temperature perturbations arising out of a single rough zone vanish away from the interface ($y\to\pm\infty$), the cumulative effect of such perturbations arising out of the array of rough zones will be noticeable far away from the interface. At the far field, this cumulative effect manifests itself in an average temperature jump γ_{av} across the interface. For the case of the periodic surface texture, the expression for the average temperature jump γ_{av} can be given by Das and Sadhal [25]

$$\gamma_{av}=\frac{1}{d}\int_{-a}^{a}\gamma(x)dx. \tag{4}$$

Because of the periodicity of all field quantities, the boundary conditions need only be considered in the interval $-d/2\leqslant x\leqslant d/2$. In the intervals $-d/2+md\leqslant x\leqslant d/2+md$, $m=\pm1,\pm2,\ldots$, the boundary conditions will be identical. The thermal boundary conditions at the interface are:

$$q_y^{+}(x,0)=q_y^{-}(x,0),\quad |x|\leqslant d/2; \tag{5}$$

$$T^{-}(x,0)-T^{+}(x,0)=R(x)q_y^{+}(x,0),\quad |x|<a;\quad T^{-}(x,0)=T^{+}(x,0),\quad a\leqslant|x|\leqslant d/2. \tag{6}$$

The mechanical boundary conditions at the interface are:

$$u_y^{+}(x,0)=u_y^{-}(x,0),\quad |x|\leqslant d/2; \tag{7}$$

$$\sigma_{yy}^{-}(x,0)=\sigma_{yy}^{+}(x,0),\quad \sigma_{xy}^{+}(x,0)=0,\quad \sigma_{xy}^{-}(x,0)=0,\quad |x|\leqslant d/2. \tag{8}$$

The boundary conditions at infinity are

$$q_x^{\infty}=0,\quad q_y^{\infty}=q^{\infty}; \tag{9}$$

$$\sigma_{yy}^{\infty}=-p^{\infty},\quad \sigma_{xy}^{\infty}=0,\quad \sigma_{xx}^{\infty}=\frac{\alpha_jE_jq^{\infty}}{K_j(1-\nu_j)}y,\quad j=1,2. \tag{10}$$

Here, $q_x(x,y)$, $q_y(x,y)$ are components of heat flux, $\sigma_{xx}(x,y)$, $\sigma_{xy}(x,y)$, $\sigma_{yy}(x,y)$ are stress components, $u_y(x,y)$ is a normal displacement, α, K, E, ν are the coefficient of linear thermal expansion, thermal conductivity, Young's modulus and Poisson's ratio, and the subscripts 1 or 2 are used to denote quantities pertaining to the lower or upper solid.

3. Solution to the problem

3.1. Reduction of the problem to a nonlinear singular integrodifferential equation

Using the technique developed by Shvets and Martynyak [26], Martynyak [32], Malanchuk et al. [37], Martynyak and Chumak [38,39] the temperature, heat fluxes, stresses and displacements in the both solids can be expressed in terms of the Muskhelishvili's complex potentials (see (15) in [39]). For the periodic contact problem under consideration this potentials has the following form:

$$F(z)=-\frac{K_{12}}{2K_jdi}\int_{-a}^{a}[\gamma(t)-\gamma_{av}]cot\frac{\pi(t-z)}{d}dt,$$

$$\Phi_1(z)=-\Phi_2(z)=\frac{K_{12}(\delta_1-\delta_2)}{2Bdi}\int_{-a}^{a}[\gamma(t)-\gamma_{av}]cot\frac{\pi(t-z)}{d}dt;$$

$$z\in S_j,\ j=1,2, \tag{11}$$