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Onset of buoyancy-driven convection in a variable viscosity liquid saturated in a porous medium

Min Chan Kim Kim

Department of Chemical Engineering, Jeju National University, Jeju 690-756, Republic of Korea

HIGHLIGHTS

- In a CO_2-IOR system, the onset of buoyancy-driven instability has been analyzed.
- Viscosity variation effect has been considered in the analysis.
- The linear stability equations were solved without the QSSA,
- Based on the linear stability results, nonlinear phenomena have been studied.

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ABSTRACT

In connection with carbon dioxide (CO_2) improved oil recovery (IOR), the onset of buoyancy-driven instability in an initially quiescent, fluid-saturated, horizontal porous layer is analyzed. Through the upper boundary, CO_2 is gradually dissolved into heavy oil to significantly reduce its viscosity but increase its density. By considering the variation of viscosity, Darcy's law is used to explain characteristics of fluid motion. Under the linear stability theory, the exchange of the stabilities is proved analytically. The critical conditions for the onset of buoyancy-driven convection are obtained as a function of the viscosity variation parameter of the Frank-Kamenetskii approximation. Based on the result of the linear stability analysis, the growth of disturbance is pursued by the direct nonlinear numerical simulation. The present linear and nonlinear analyses show that the viscosity variation parameter plays a critical role in the onset and the growth of the instability motion.

1. Introduction

The carbon dioxide (CO_2) improved oil recovery (IOR) has been applied in petroleum production since the first commercial CO_2-IOR project began operation in 1972 (Sweatman et al., 2009). In the CO_2-IOR system, CO_2 is gradually dissolved into heavy oil to significantly reduce its viscosity. Consequently, the CO_2-saturated heavy oil is mobile enough to enhance the recovery of petroleum. It has been known that in the US alone, CO_2 injection has accounted for the recovery of about 1.5 billion barrels of oil (Ahmed et al., 2012). Furthermore, CO_2 injection into oil reservoir may reduce the global warming through the CO_2 sequestration. Therefore, better understanding of CO_2 injection mechanism is critical in the design and operation of the CO_2-IOR process.

From the published data of the viscosity and density of hydrocarbons, at a certain condition, the CO_2 dissolution induces the increase of oil density and the decrease of its viscosity (Ahmed et al., 2012). Therefore, the CO_2 injection can be accelerated through the buoyancy-driven convection which occurs in a fluid-saturated porous medium due to adverse density gradient. In the CO_2-IOR system, increasing CO_2 concentration from above creates an unstable density profile and induces the buoyancy-driven convection. In this case, the heavier CO_2 saturated oil will flow downward and will be replaced by oil with lesser CO_2 content. Since the density increase due to the CO_2 dissolution is around 10% at 11.7 MPa and 46.7 °C, this can lead to a convective mixing process, which significantly accelerates the dissolution of CO_2, and thus improves the recovery process (Ahmed et al., 2012). It has been well-known that this buoyancy-driven convection can be also occurred in CO_2 dissolution in the saline water (Ennis-King et al., 2005).

The analysis of convective instability in a porous medium begins with Horton–Rogers–Lapwood (HRL) convection (Horton and Rogers, 1945; Lapwood, 1948). They examined thermally driven convection and used the methods developed for convection in a homogeneous fluid. The extension of the classical HRL convection was well summarized by Nield and Bejan (2013). Recently, Rajagopal et al. (2011) studied the variable viscosity on the onset of buoyancy-driven convection. In the above mentioned

E-mail address: mckim@cheju.ac.kr

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studies, it was assumed that there was a linear increase in temperature with depth, appropriate for gradual heating or for a steady state, e.g., the naturally occurring geothermal gradients in the subsurface. However, in many experimental situations and field studies, there is a relatively rapid change in temperature or solute concentration at one boundary. The basic profile of temperature or concentration before the onset of convection is then time dependent. The related instability analysis has been conducted by using the quasi-steady state approximation (QSSA) (Caltagirone, 1980), and the initial value problem approaches (Ennis-King et al., 2005). Also, the stability of time dependent base states has been investigated by energy method (Ennis-King et al., 2005) and its modification (Kim and Choi, 2007). Recently, Riaz et al. (2006) analyzed the onset of convection in a porous medium under the time-dependent concentration field in a self-similar coordinate. They employed the dominant mode analysis, the quasi-steady state approximation (QSSA) and direct numerical simulation. They showed that the QSSA in the self-similar coordinate provides quite accurate results. Rapaka et al. (2008, 2009) considered the maximum amplification over all possible infinitesimal perturbations introduced at the same time and compared their results with those form the linear amplification theory which has been used widely (Ennis-King et al., 2005; Hassanzadeh et al., 2006; Slim and Ramakrishnan, 2010). Very recently, Kim and Choi (2012) conducted systematic analysis on the onset of gravitational fingering instability with and without the QSSA. However, all the above mentioned work has been done for the constant viscosity fluid.

In the present study, taking CO_2 dissolution in oil into account, the onset and growth of buoyancy-driven instability is analyzed theoretically. The present system seems to be quite similar to the CO_2 sequestration system, where CO_2 is partially miscible in water with a low saturation concentration. However, the physical properties of the CO_2/water system do not depend strongly on the CO_2 concentration due to the low solubility of CO_2 in water. Therefore, there are some important differences between CO_2/oil and CO_2/water systems. The present CO_2/oil system shows a concentration-dependent density and viscosity relationships (Ahmed et al., 2012). So, the concentration-dependent viscosity effects should be considered in theoretical and experimental studies. In the present study the onset of buoyancy-driven convection in a porous medium saturated by a variable viscosity liquid is investigated by using the linear and nonlinear analyses. The new stability equations are re-derived in the semi-infinite domain and finite one, and solved without the QSSA. Through the initial growth rate analysis, the most dangerous mode of disturbance is identified. The critical conditions for the onset of convection are obtained as a function of the Darcy-Rayleigh number and the viscosity variation parameter.

2. System and governing equations

The system considered here is an initially quiescent, fluid-saturated, horizontal porous layer of depth d, as shown in Fig. 1. Initially the fluid layer contains no solute, i.e. $C=0$ at $t=0$. For time $t \geq 0$, gas starts to dissolve into fluid layer trough the upper boundary which is assumed to be maintained at uniform concentration C_i. The lower boundary is assumed to be impermeable and no mass flux condition. The standard governing equations for solute-driven convective mixing consist of Darcy's law for the fluid motion in a porous medium and the convective diffusion equation for the transport of the dissolved solute. Under the Boussinesq approximation, they can then be written as follows (Caltagirone, 1980):

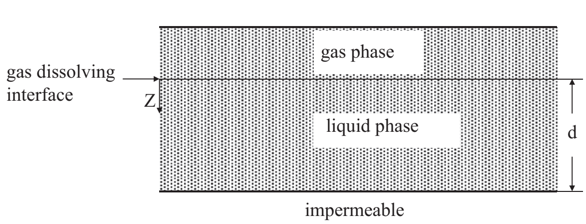

Fig. 1. Schematic diagram of system considered here.

$$\nabla \cdot \mathbf{U} = 0, \tag{1}$$

$$\frac{\mu}{K}\mathbf{U} = -\nabla P + \beta \mathbf{g} C, \tag{2}$$

$$\varepsilon \frac{\partial C}{\partial t} + \mathbf{U} \cdot \nabla C = \mathbf{D} \nabla^2 C, \tag{3}$$

where $\mathbf{U}$ is the Darcy velocity vector, μ is the fluid viscosity, K is the permeability, P is the pressure, $\mathbf{g}$ is the gravitational acceleration, ε is the porosity C is the solute concentration, t is time, β is the volumetric expansion coefficient and $\mathbf{D}$ is the effective diffusivity of the solute in the liquid phase saturated in the porous medium. In the present study, the viscosity is assumed to be having following form:

$$\frac{\mu}{\mu_r} = \overline{\mu}(c), \tag{4}$$

where $c = C/C_i$ and μ_r is the viscosity at $C=0$. Darcy's equation (2) with the variable viscosity model, Eq. (4) has been widely used in the similar problem (Homsy, 1987). The important parameters to describe the present system are the Darcy-Rayleigh number Ra defined by

$$Ra = \frac{g\beta K C_i d}{\mathbf{D}\nu_0}.$$

For a system of large Ra, the stability problem becomes transient and very difficult, and the critical time t_c to mark the onset of buoyancy-driven motion remains unsolved. For this transient stability analysis, a set of nondimensionalized variables z, τ and c_0 is defined by using the scale of vertical length $L_c(=d/Ra)$, time $L_c^2/(\varepsilon \mathbf{D})$, and concentration C_i. If the effect of volume expansion of the liquid layer due to the CO_2 dissolution can be neglected, the basic conduction state is represented in dimensionless form by

$$\frac{\partial c_0}{\partial \tau} = \frac{\partial^2 c_0}{\partial z^2}, \tag{5}$$

with the following initial and boundary conditions,

$$c_0 = 0 \quad \text{at } \tau = 0, \tag{6a}$$

$$c_0 = 1 \quad \text{at } z = 0 \quad \text{and} \quad \frac{\partial c_0}{\partial z} = 0 \quad \text{at } z = Ra, \tag{6b}$$

The above equations can be solved by using conventional separation of variables technique or Laplace transform method as follows (Kim and Choi, 2012):

$$c_0(\tau, z) = 1 - \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{(2n-1)} \sin\left\{\left(\frac{(n-1/2)\pi}{Ra}\right) z\right\} \exp\left\{-\left(\frac{(n-1/2)\pi}{Ra}\right)^2 \tau\right\}, \tag{7a}$$

$$c_0(\tau, \zeta) = \sum_{n=0}^{\infty} (-1)^n \left\{ erfc\left(\frac{\zeta}{2} + \frac{n}{\sqrt{\tau}} Ra\right) + erfc\left(\frac{n+1}{\sqrt{\tau}} Ra - \frac{\zeta}{2}\right)\right\}, \tag{7b}$$

where $\zeta = z/\sqrt{\tau}$. Eq. (7b) converges more rapidly than Eq. (7a) for a small time region. If the penetration depth $\delta(\propto \sqrt{\tau})$ over which c_0 is non-zero is much smaller than the extent of the system, i.e.$(\sqrt{\tau}/Ra) \ll 1$, the domain can be considered semi-infinite in the positive z-direction, and Eq. (7) can be reduced as

$$c_0 = \text{erfc}\left(\frac{\zeta}{2}\right), \tag{8}$$