Chemical Engineering Science 111 (2014) 191–202

Contents lists available at ScienceDirect

Chemical Engineering Science

journal homepage: www.elsevier.com/locate/ces

Determination of head change coefficients for dividing and combining junctions: A method based on the second law of thermodynamics

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HIGHLIGHTS

- By a second law analysis for branched flows head change coefficients are determined.
- The often neglected energy transfer between partial flows is taken into account.
- Engulfment of partial flows and its initiation in numerical solutions is analyzed.
- CFD solutions for various examples may serve as benchmark solutions.

ARTICLE INFO

Article history:
Received 27 November 2013
Received in revised form
4 February 2014
Accepted 25 February 2014
Available online 3 March 2014

Keywords:
T-junction
T-mixer
Engulfment
Entropy generation
Stress work rates
Micro-flow

ABSTRACT

Losses due to the flow through conduit components in a pipe system can be accounted for by head loss coefficients *K*. They correspond to the dissipation in the flow field or, in a more general sense, to the entropy generation due to the conduit component under consideration. When only one single mass flow rate is involved, an entropy based approach is straight forward since the flow rate can be used as a general reference quantity. If, however, one mass flow rate is split or two partial flow rates come together like in junctions, a new aspect appears: there is an energy transfer between the single branches that has to be accounted for. It turns out that this energy transfer changes the total head in each flow branch in addition to the loss of total head due to entropy generation. Therefore, appropriate coefficients for junctions should be named as head change coefficients. As an example, the method is applied to laminar flows. Head change coefficients for dividing and combining flows in a T-shape micro-junction are determined for both branches and discussed with respect to their physical meaning. For the combining junction, the special case of engulfment, leading to enhanced mixing in micro-mixers, is also considered. Finally, it is shown, how the newly defined coefficients can be used for the design of a flow network.

1. Introduction

In standard text books, e.g. Munson et al. (2005), loss coefficients are introduced as a non-dimensional pressure difference

$$K_{\mathrm{p}} = \frac{\Delta p}{\rho u_{\mathrm{m}}^2/2}. \tag{1}$$

A pressure difference Δp between two cross sections ① and ②, however, is only equivalent to a loss of mechanical energy, when the kinetic and potential energies in both cross sections are unchanged. For the general case including an acceleration of the flow, e.g. due to different areas of the cross sections, the head loss coefficient should be based on the specific dissipation φ of mechanical energy

$$K = \frac{\varphi}{u_{\mathrm{m}}^2/2}. \tag{2}$$

The specific dissipation is the conversion of mechanical energy into internal energy. This process is accompanied by entropy generation, see the following section. For incompressible one-dimensional flows φ can be determined based on the so-called extended Bernoulli-equation

$$\varphi_{12} = \frac{p_1 - p_2}{\rho} + \frac{\alpha_1 u_{\mathrm{m}1}^2 - \alpha_2 u_{\mathrm{m}2}^2}{2} + g(y_1 - y_2) \tag{3}$$

with $\alpha_i u_{\mathrm{m}i}^2/2$ as the specific kinetic energy in a cross section ⓘ and $g y_i$ as the specific potential energy. This procedure follows the so-called *indirect approach*. In simple experiments, however, the exact value of the kinetic energies cannot be determined. This is why the kinetic energy is often approximated as $u_{\mathrm{m}i}^2/2$, i.e. $\alpha_i = 1$ like

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http://dx.doi.org/10.1016/j.ces.2014.02.035

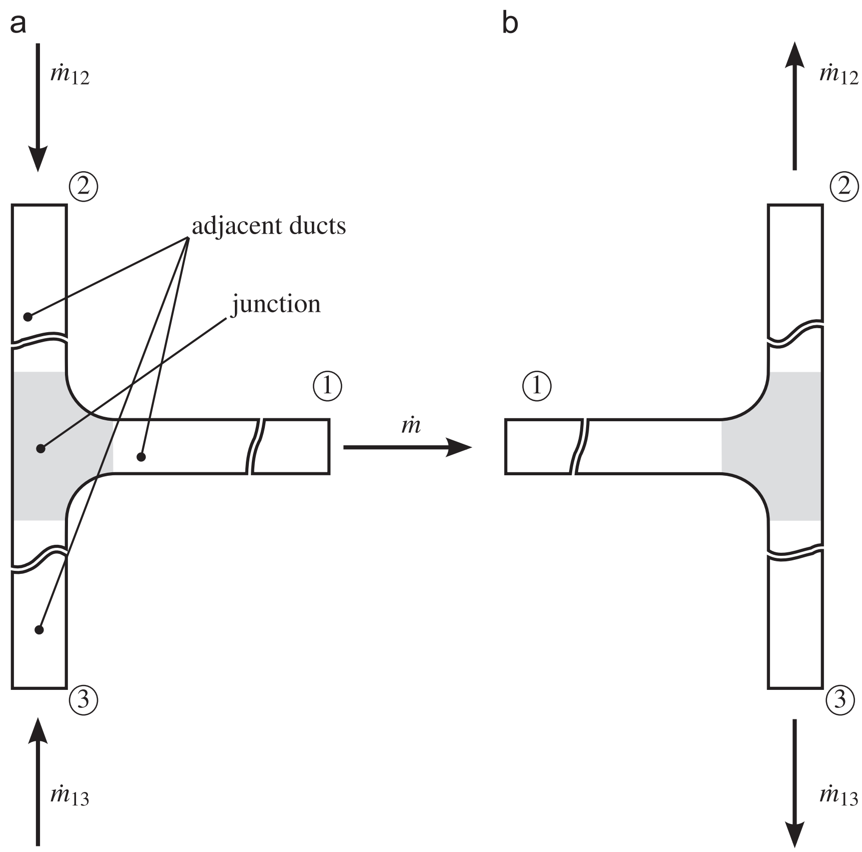

Fig. 1. Combination or division of flow rates in junctions. (a) Combining junction and (b) dividing junction.

in Miller (1978), for example. However, for conduit components with extremely low losses and large changes of kinetic energy like in a nozzle, this procedure can lead to very wrong results for the dissipation, see Schmandt and Herwig (2011a).

Another shortcoming of the indirect approach occurs when branched flows are considered. For the case of a dividing junction, a flow rate $\dot{m}$ enters the junction at cross section ① and leaves it as partial flow rates $\dot{m}_{12}$ and $\dot{m}_{13}$ at the respective cross sections ② and ③, see Fig. 1(b). Since both flow rates are in direct contact, an energy transfer occurs. For steady flows, a stream surface (interface) can be identified over which energy is transferred by viscous forces. In the Bernoulli equation, this can be expressed as a specific diffusion rate (or stress work rate) d.

$$\varphi_{12} - d_{12} = \frac{p_1 - p_2}{\rho} + \frac{\alpha_{1;12} u_{m1}^2 - \alpha_2 u_{m2}^2}{2} + g(y_1 - y_2) \tag{4}$$

$$\varphi_{13} - d_{13} = \frac{p_1 - p_3}{\rho} + \frac{\alpha_{1;13} u_{m1}^2 - \alpha_3 u_{m3}^2}{2} + g(y_1 - y_3) \tag{5}$$

The mutual energy transfer rates are linked by the first law of thermodynamics, i.e.

$$d_{12}\dot{m}_{12} = -d_{13}\dot{m}_{13} \tag{6}$$

Existing definitions of K for junctions, see e.g. Miller (1978), Serre et al. (1994), Sharp et al. (2010), Ramamurthy et al. (2006) are based on the "head loss" $\Delta H = (\varphi_{ij} - d_{ij})/g$, which better should be called a head change, see Schmandt and Herwig (2013). An energy transfer between the branches is already mentioned in Miller (1978), its contribution to the head change, however, cannot be determined based on the indirect approach.

In the following section, a method will be introduced, which permits the determination of a head change coefficient

$$K_{ij} = \frac{\varphi_{ij} - d_{ij}}{u_m^2/2}. \tag{7}$$

based on the contributions of φ_{ij} and d_{ij} within K_{ij}. Here u_m is the mean velocity in the reference cross section (here: ①).

2. The SLA-approach for branched flows

In order to introduce our general concept to account for a head change, we first discuss unbranched flows. As already mentioned, the specific dissipation

$$\varphi = \dot{\Phi}/\dot{m} \tag{8}$$

for unbranched flow is accompanied by an entropy generation. Generally entropy generation occurs in the flow and temperature fields and thus can be determined as a scalar field variable when the velocity and the temperature fields from a converged CFD simulation are available. Using the entropy generation and thus the second law of thermodynamics for the characterization of processes is called second law analysis (SLA).

The local entropy generation rate due to dissipation alone (in $\mathrm{W\,m^{-3}\,K}$) can be computed as

$$\dot{S}''' = \frac{\overrightarrow{\overrightarrow{\tau}} : \nabla \overrightarrow{u}}{T} \tag{9}$$

leading to

$$\dot{S}''' = \frac{\mu}{T}\left(2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]\right.$$