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Modelling of quasi-static adiabatic bubble formation, growth and detachment for low Bond numbers

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HIGHLIGHTS

- Geometric model for low Bond number applications is presented.
- A corresponding geometric detachment relation is developed.
- A force instability criterion is developed for the onset of detachment.
- A corresponding adiabatic bubble growth model is developed.
- The model is validated with a numerical treatment to the problem.

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ABSTRACT

In an effort to lessen the computational expense of bubble growth simulations without compromising its fundamental shape characteristics, an analytical model is developed. It is substantiated using validated numerical results simulating quasi-static adiabatic bubble growth for Bond numbers less than 0.07 in which its characteristic length is the radius of the cavity from which the bubble is issuing. The model's ability to predict shape and size evolution for bubble formations is shown to predict the growth and detachment volume to be in the range from 0.05% for a 0.00137 Bond number to 3% for a 0.06032 Bond number.

The model builds upon a recent numerical study which showed that the shape evolution of a quasi-static bubble formation may be idealised as a spherical segment atop a cylindrical neck for low Bond number applications. By incorporating this geometry, the present work's proposed model accounts for bubble shape transformation throughout the bubble growth cycle by including a necking phenomenon in which the bulk of the bubble rises due to an elongating base as it prepares to detach. This is accomplished by introducing: (1) a volume condition which geometrically relates the neck height with the bubble's spherical segment at detachment; (2) a force instability criterion signalling the onset of detachment which relates the size of the bubble to its Bond number and cavity radius; and (3) a neck evolution growth curve. The analytical model ties these relations together with the use of the characteristics of the proposed geometry generating a full description of quasi-static adiabatic bubble growth and detachment for low Bond number formations. The resulting predicted bubble growth characteristics, such as profile, volume, centre of gravity, truncated sphericity and aspect ratio, are presented and discussed with respect to a validated numerical treatment of the problem.

1. Introduction

Bubble formation, growth and detachment events are commonplace in many industrial processes and technologies such as bubble column reactors (e.g., Wu and Gidaspow, 2000), power generation and mineral processing (e.g., Witt et al., 1998) and ink-jet printing (e.g., Van Dam and Le Clerc, 2004) among others. The physical nature of the problem has been the subject of many analytic (e.g., Plesset and Zwick, 1954; Forster and Zuber, 1954; Scriven, 1959; Mikic and Rohsenow, 1969; Riznic et al., 1999), numerical (e.g., Sparks, 1978; Yoon et al., 2001; Robinson et al., 2010; Albadawi et al., 2013) and experimental investigations (e.g., Van Stralen et al., 1975; Ma et al., 2012; Di Bari et al., 2013; Lesage and Marois, 2013).

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A primary motivation for this area of research is to gain deeper understanding of the physical phenomena which can subsequently be used to advance the state of the art by engineering higher performance technologies and systems.

Adiabatic bubble growth and detachment from submerged orifices has been studied for many decades for which a detailed review is provided in Kulkarni and Joshi (2005). Recently, the state of the art has advanced considerably as CFD models have been used to predict bubble growth and detachment characteristics and, by solving the velocity field in the liquid, have provided detailed information with regard to the complex interaction between the phases and the deformable interface as the bubbles grow and detach (Gerlach et al., 2007; Quan and Hua, 2008; Das and Das, 2009). Other more simplified numerical treatments of the problem have been performed by assuming potential flow and using boundary integral methods to facilitate the prediction of the bubble shape without requiring the full solution to the Navier–Stokes equations (Oguz and Prosperetti, 1993; Xiao and Tan, 2005).

The preponderance of the work in the open literature relate to mechanistic models which are posed to predict or provide insight into bubble growth and detachment. These models must make geometric simplifications regarding the bubble shape. Generally the bubbles are assumed to be a perfect sphere or a section of a sphere (Davidson and Schuler, 1960; Kupferberg and Jameson, 1969; LaNauze and Harris, 1974; Buyevich and Webben, 1996; Zhang and Tan, 2000; Duhar and Colin, 2006). In order to gain sufficient physical insight without having to solve the velocity field, the bubble growth problem is typically formulated by considering the net upward and downward forces acting on the bubble and either balancing these forces (Zhang and Tan, 2000) or equating the unbalanced forces to the rate of change of momentum of the bubble (Duhar and Colin, 2006). However, these mechanistic models generally require *a priori* information about the bubble, in the sense that the forces are quantified subsequent to obtaining the measurements (Loubière and Hébrard, 2003; Duhar and Colin, 2006) and almost always the bubble is assumed to detach as a perfect sphere (Gaddis and Vogelpohl, 1986; Nahra and Kamotani, 2000; Thorncroft et al., 2001; Duhar and Colin, 2006). However, the force model is required to attribute a force due to surface tension to a bubble surface contact which deforms the bubble from spherical. Otherwise, an unrealistic singularity as a point of contact is assumed nullifying any force due to surface contact.

The present work alleviates the computational expense associated with solving a full velocity field model without compromising the analysis of the two fundamental forces at play in quasi-static bubble formations: buoyancy and surface tension. The bubble cap is assumed to remain a section of a sphere and a cylindrical neck is permitted to form beneath the bubble cap until a detachment criterion is achieved. In this way, the buoyancy force and force due to surface tension may be calculated from the same geometric model. The main thesis proposed is that for quasi-static bubble growth the linear momentum of the bubble is, by virtue of being quasi-static, negligibly small. This scenario is considered in the present work to be somewhat paradoxical since it is also true that the bubble centre of gravity moves which cannot be accounted for from the traditional expression relating the velocity of the centre of mass to the momentum of the object. However, the centre of gravity of the bubble can also move if there is an asymmetric addition of mass to the system. This is the case for a bubble emerging from an orifice where the triple contact line remains fixed to the rim of the orifice. It is quite obvious that the opposing influence of surface tension and buoyancy cause a neck to form and elongate beneath the bubble which will also influence the motion of the bubble centre of gravity. This problem has been formulated separately, along with a force instability criterion in which a transition to a net force upwards initiates a topological change from a single attached bubble to a detached bubble and a smaller residual bubble at the cavity. The analytic expressions developed include: (1) a detachment condition based on mass and geometric constraints; (2) a detachment condition based on a force instability criterion; and (3) a bubble neck growth curve based on the bubble's degree of truncated sphericity. These expressions complete the bubble cycle from inception to detachment and are validated against a host of experimental results and validated numerical simulations of the problem.

2. Geometric model

2.1. Geometric assumptions

A common shortcoming of analytical attempts to describe bubble growth and/or bubble detachment stem from geometric models which over constrain the shape of the bubble as it transitions from inception to detachment. Furthermore, it is common to exclude the nucleation site characteristics from the modelling of a growing and detaching bubble and to describe its growth using the Bond number with characteristic length equal to the bubble radius (Fritz, 1935; Zuber, 1959; Cole, 1967; Kutateladze and Gogonin, 1980). In the present work, the bubble growth and detachment characteristics are discussed in terms of the Bond number Bo_b, or equivalently the Eötvös number (Eo), with characteristic length equal to the cavity radius in order to include a cavity characteristic in the analysis. The Bond number is therefore defined as

$$Bo_b = \frac{\Delta\rho g b^2}{\sigma} \tag{1}$$

in which b is the cavity radius, g is the gravitational constant, σ is the surface tension and $\Delta\rho$ is the difference in densities of the two phases. It is important to note that in this way, the Bond number represents the square of the ratio of the orifice radius to the Capillary length defined as

$$L_c = \sqrt{\sigma\ \Delta\rho^{-1} g^{-1}}. \tag{2}$$

In the present investigation, a geometric model is proposed in which a quasi-static bubble evolves from a hemisphere into a truncated sphere with a fixed base radius while rising due to the formation and elongation of a cylindrical neck at its base.

The key assumptions are:

i. Early in the growth cycle the bubble attains a hemispherical shape. This is due to the fact that in this early stage, buoyancy is not a dominant force and the bubble will take a hemispherical shape in order to minimise its free energy.
ii. The bubble will grow as a spherical segment rising due to an elongating cylindrical neck with a radius equal to the orifice radius.
iii. Subsequent to detachment a mass equivalent to that of a hemispherical bubble is left at the orifice.

The model's assumptions are based on the numerical results obtained by Lesage et al. (2013) in their analysis of bubble shape evolution. In their work, experimentally validated treatments of the Capillary equation (Mori and Baines, 2001; Gerlach et al., 2005; Lesage and Marois, 2013) show that low Bond number applications experience no inward pinching at the neck of the bubble throughout its growth cycle. In their quasi-static bubble detachment study Lesage and Marois (2013) observed that this phenomenon is due to a hydrostatic pressure term that is inferior