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# Free-surface shape in unbaffled stirred vessels: Experimental study via digital image analysis

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## HIGHLIGHTS

- Free surface measurement in uncovered unbaffled stirred vessel via image analysis.
- Development of an original two parameters model for free-surface description.
- Robust fitting of model equation to free surface profile by means of a self-developed algorithm.
- Free-surface data relevant to several vessel geometries are presented and discussed.
- Data presented provide useful design information and viable benchmark data for CFD validation.

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## ABSTRACT

There is a growing interest in using unbaffled stirred tanks for addressing a number of processing needs such as low shear damage (sensitive biocultures), low attrition (solid–liquid applications), deep-cleaning/sterilization (pharmaceutical applications). The main feature of uncovered, unbaffled stirred tanks is highly swirling motion of the fluid that results in a deformation of the free liquid surface. At sufficiently high agitation speeds the resulting whirlpool reaches the impeller and gives rise to a gas–liquid dispersion, so leading to the formation of a dispersion without the use of gas-sparger; the so-called *self-inducing* operation of the vessel.

In this work, digital image analysis coupled with a suitable shadowgraphy-based technique is used to investigate the shape of the free-surface that forms in uncovered unbaffled stirred tanks, when different stirrer geometries are considered. The technique is based on back-lighting the vessel and suitably averaging over time the recorded free surface shape.

For each investigated geometry, the deformed free-surface was analyzed at different impeller speeds. Different geometries of the vessel were analyzed, by varying impeller distance from vessel bottom as well as agitator type (Rushton turbine, Lightnin A310, Pitched Blade Turbine). It is shown that impeller design strongly affects the free surface profile, and in turn the impeller speed at which the free surface reaches the impeller. A model was developed to fully describe free-surface profile at all agitation speeds and for all investigated geometries, suitable for being adapted to experiments by means of physically consistent parameters adjustment.



## 1. Introduction

The main feature of unbaffled stirred tanks is the strongly swirling liquid motion, that due to centrifugal effects leads to the formation of a vortex on the liquid free-surface. Also, impeller–liquid relative velocities are rather small, so resulting in a comparatively small pumped flow rate and in turn in poorer axial mixing with respect to baffled tanks. The adoption of baffles effectively destroys the circular liquid patterns, hence inhibits main vortex formation and strongly increases the pumped flow rate, leading to improved mixing rates. For these reasons baffles are almost invariantly adopted in stirred tanks.

There are cases, however, in which the use of unbaffled tanks may be desirable. Baffles are usually omitted in the case of very viscous fluids ($Re < 20$) as they can give rise to dead zones that are bound to worsen the mixing performance (Nagata, 1975). Unbaffled tanks may be advisable in crystallizers, where baffles may promote particle attrition (Mazzarotta, 1993), and give rise to higher fluid–particle mass transfer rates for a given power consumption (Grisafi et al., 1998;

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Brucato et al., 2010), even if baffles promote solids suspension and circulation. Vortex existence may be useful when floating particles are to be drawn down (Freudig et al., 1999) or when gas bubbles have to be rapidly removed from the liquid (Smit and During, 1991). As concerns bioreactor applications, for aerobic fermentations and plant or animal cell cultivations liquid agitation is required in order to ensure oxygen and nutrient transfer and to maintain cells in suspension. When shear sensitive cells are involved, both mechanical agitation and sparging aeration can cause cell death, which makes it advisable the adoption of unbaffled, unsparged vessels (Scargiali et al., 2012b). In addition, unbaffled stirred vessels can be operated as self-ingesting reactors, showing interesting characteristics (Scargiali et al., 2012b), even when compared with both sparged (Scargiali et al., 2010) and self-ingesting (Conway et al., 2002; Scargiali et al., 2012a) baffled stirred vessels. Clearly, in all these cases the ability of correctly predicting free surface shape plays an important role in the vessel design.

The whirlpool in unbaffled mixing tanks is formed in the central part of the vessel as a result of centrifugal forces acting on the rotating liquid, hence its shape mainly depends on the flow field. Some of the works dealing with unbaffled tank modelling (Nagata, 1975; Smit and During, 1991) pointed out that liquid tangential velocity mainly depends on the distance from the shaft. Axial variation is small, with the exception of the zone near vessel bottom, where the tangential velocities are decreased by bottom wall friction. On this basis, a simplified potential flow model can be adopted for vortex geometry description (Nagata, 1975; Rieger et al., 1979; Smit and During, 1991; Ciofalo et al., 1996). In fact, by assuming that velocity in the vessel is purely tangential ($u_r = u_y = 0$) and only depends on the radial coordinate ($u_\theta = u_\theta(r)$), Navier–Stokes equations in cylindrical coordinates (neglecting viscous forces) reduce to:

$$\begin{cases} \dfrac{\partial p}{\partial r} = \rho \dfrac{u_\theta^2}{r} \\ \dfrac{\partial p}{\partial z} = -\rho g \end{cases} \tag{1}$$

At the liquid surface $h$ pressure is constant ($p(r,z) = p_0$), hence its derivative must be identically nil:

$$dp = \frac{\partial p}{\partial r} dr + \frac{\partial p}{\partial z}\, dz = 0$$

By substituting Eq. (1), the following differential equation is obtained describing vortex profile as a function of the radial coordinate:

$$\frac{dh}{dr} = \frac{u_\theta^2}{rg} \tag{2}$$

Hence, the free-surface profile $h(r)$ can be derived once $u_\theta(r)$ is known. Eq. (2) may be suitably made dimensionless by introducing the following dimensionless quantities:

$$\theta = \frac{u_\theta}{U_{tip}} = \frac{u_\theta}{\omega D/2} = \frac{u_\theta}{\pi D N}$$

$$\xi = \frac{r}{D/2}$$

$$\psi = \frac{h}{D}$$

$$Fr = \frac{N^2 D}{g}$$

the following general dimensionless equation for the description of free-surface profile is obtained:

$$\frac{d\psi}{d\xi} = \pi^2 Fr \frac{\theta^2}{\xi} \tag{3}$$

To determine free-surface profile, Nagata (1975) suggested that the whole flow field can be subdivided into an inner region $\xi \le \xi_c$ (forced vortex region) exhibiting a rigid body motion with angular velocity $\omega$ and an outer region $\xi > \xi_c$ (free vortex region) where the angular momentum $u_\theta r$ is constant. This leads to the following tangential velocity profile:

$$u_\theta = \begin{cases} \omega r & \text{if } r \le r_c \\ \dfrac{\omega r_c^2}{r} & \text{if } r > r_c \end{cases} \tag{4}$$

In a dimensionless form:

$$\theta = \begin{cases} \xi & \text{if } \xi \le \xi_c \\ \dfrac{\xi_c^2}{\xi} & \text{if } \xi > \xi_c \end{cases} \tag{5}$$

By substituting Eq. (5) into Eq. (3), integrating and imposing the continuity of free-surface profile, the following equation can be obtained as a function of the liquid level at the vessel wall $\psi_w$:

$$\psi = \begin{cases} \psi_w - \dfrac{\pi^2 \xi_c^2}{2}\left(2 - \dfrac{\xi_c^2}{\xi_w^2} - \dfrac{\xi^2}{\xi_c^2}\right) & \text{if } \xi \le \xi_c \\ \psi_w - \dfrac{\pi^2 \xi_c^2}{2}\left(\dfrac{\xi_c^2}{\xi^2} - \dfrac{\xi_c^2}{\xi_w^2}\right) & \text{if } \xi > \xi_c \end{cases} \tag{6}$$

where $\xi_w = T/D$ and $\xi_c = 2r_c/D$. By imposing total liquid mass conservation, liquid levels at the vessel wall and at the vessel axis $\psi_b$ can be written as a function of the liquid level at rest $\psi_0$:

$$\psi_w = \psi_0 + \frac{\pi^2 \xi_c^2}{2} \frac{\xi_c^2}{\xi_w^2}\left(\frac{1}{2} - \ln \frac{\xi_c^2}{\xi_w^2}\right) \tag{7}$$

$$\psi_b = \psi_0 + \frac{\pi^2 \xi_c^2}{2} \frac{\xi_c^2}{\xi_w^2}\left(\frac{3}{2} - \ln \frac{\xi_c^2}{\xi_w^2} - 2\frac{\xi_c^2}{\xi_w^2}\right) \tag{8}$$

By simple manipulation the following expression for overall vortex depth is obtained:

$$\Delta\psi_v = \psi_w - \psi_b = \frac{\pi^2 \xi_c^2}{2}\left(2 + \frac{\xi_c^2}{\xi_w^2}\right) Fr \tag{9}$$

In this model, the only free parameter is the dimensionless radius $\xi_c$ that was found from vortex depth measurements to be in the range 0.55–0.65 for systems agitated by Rushton turbines with $D/T = 0.3-0.7$ and $Re = \rho N D^2/\mu$ above $4 \times 10^4$ (Nagata, 1975).

Smit and During (1991) investigated vortex shape in an unbaffled tank equipped with 4 pitched-blade turbine having $D/T = 0.8$, finding that for $Fr < 0.4$, $\Delta\psi_v = 3.33Fr$. On the basis of LDA measured tangential velocities these authors suggested that the following velocity profile has to be adopted for free surface profile calculation:

$$u_\theta = \begin{cases} 0.825\omega r & \text{if } r \le r_c \\ 0.825 r_c \left(\dfrac{\omega r_c}{r}\right)^{0.6} & \text{if } r > r_c \end{cases} \tag{10}$$

and in dimensionless form:

$$\theta = \begin{cases} 0.825\xi & \text{if } \xi \le \xi_c \\ 0.825\xi_c \left(\dfrac{\xi_c}{\xi}\right)^{0.6} & \text{if } \xi \le \xi_c \end{cases} \tag{11}$$

where $\xi_c$=0.675.

This is significantly different from Nagata's model, as for the forced-vortex-region rigid body motion is still assumed but with a revolution frequency smaller than $N$, while in the free vortex region a velocity profile not obeying to angular momentum conservation is adopted, *i.e.* the product $v_\theta r^{0.6}$ is kept constant instead of $v_\theta r$. Notably, these velocity profiles are relevant to a