Chemical Engineering Science 102 (2013) 200–208

Contents lists available at ScienceDirect

Chemical Engineering Science

journal homepage: www.elsevier.com/locate/ces

Drag reduction induced by polymer in turbulent pipe flows

Shu-Qing Yang, Donghong Ding *

Faculty of Engineering, University of Wollongong, Northfields Ave, Wollongong, NSW 2522, Australia

HIGHLIGHTS

- Equations of the velocity distribution and friction factor in turbulent drag reducing flows are derived.
- The approach of predicting the onset Reynolds number for drag reduction has been provided.
- The approach of determining the optimal polymer concentration for the Maximum Drag Reduction is obtained.

ARTICLE INFO

Article history:
Received 3 March 2013
Received in revised form
22 July 2013
Accepted 28 July 2013
Available online 9 August 2013

Keywords:
Fluid mechanics
Drag reduction
Turbulence
Polymers
Maximum drag reduction
Mathematical model

ABSTRACT

This paper deals with the velocity distribution and friction factor of polymer drag reducing flows in smooth pipes. By applying the concept of elastic shear stress or the Reynolds shear stress deficit, the equations of velocity distribution and friction factor in turbulent drag reducing flows are derived. Based on the derived equations, the onset Reynolds number for drag reduction is discussed which shows that the onset of drag reduction depends on the polymer type and its concentration. The optimal polymer concentration for Maximum Drag Reduction (MDR) is obtained, and it depends only on the polymer species. The effect of polymer mechanical degradation is discussed in the current paper.

1. Introduction and background

The addition of a minute amount of long-chain flexible polymer molecules to flowing fluids can drastically reduce turbulent friction, also known as drag reduction, which was initially observed by Toms (1948). Drag reduction has subsequently attracted extensive research due to its practical applications and fundamental importance (Lumley, 1969; Virk, 1975; Toonder et al., 1997; White, 2008). Various experimental and numerical works from the fields of chemistry, physics, and fluid mechanics have been conducted to understand how the polymer behaves and why drag reduction occurs. However, due to the complexity of the problem, the exact mechanism for the drag reduction is still poorly understood. Experiments indicated that there are three identifiable phenomena in drag reducing flows, i.e. the onset phenomenon, the Maximum Drag Reduction (MDR) and the "shear deficit" or the additional polymer shear stress.

The onset of drag reduction is identified by a certain threshold Reynolds number, below which the flow behaves like a Newtonian solvent and yields no drag reduction. The flow resistance is similar to that of solvent in the absence of additives and follows the well-known Prandtl–von Karman relationship, i.e.,

$$\frac{1}{\sqrt{f}} = 4.0\log_{10}(Re\sqrt{f}) - 0.4, \tag{1.1}$$

where Re is the pipe Reynolds number, i.e. $2Vr/\nu$. V is the mean velocity, ν is the kinematic viscosity of the fluid and r is the pipe radius. Fanning's friction factor, f, is defined as

$$f = 2\left(\frac{u_*}{V}\right)^2, \tag{1.2}$$

where u_* is the friction velocity. The velocity profile can be approximately expressed by

$$u^+ = y^+ \quad y^+ < 11.6, \tag{1.3a}$$

$$u^+ = 2.5\ln y^+ + 5.5 \quad y^+ \geq 11.6, \tag{1.3b}$$

where u^+ is the normalized velocity, i.e., u/u_*, u is the local velocity; y^+ is the normalized distance from a wall. i.e., $u_* y/\nu$ and y is the distance from the wall. Eq. (1.3a) is valid in the viscous

* Corresponding author. Postal address: Building 6, University of Wollongong, NSW 2522, Australia. Tel.: +61 4 52420 348.
E-mail address: dd443@uowmail.edu.au (D. Ding).

0009-2509/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ces.2013.07.048

sublayer and the applicability of Eq. (1.3b) corresponds to the turbulent core.

When the Reynolds number is larger than the onset threshold, the drag reduction occurs. For this case Virk (1971a) found that the log-law is shifted by an amount ΔB in turbulent core, with no change of slope, and the velocity distribution can be expressed by

$$u^+ = 2.5\ln y^+ + 5.5 + \Delta B. \tag{1.4}$$

On the other extreme, experiments showed that a dilute polymer solution has a state of Maximum Drag Reduction (MDR). Virk (1971b) observed that there exists a universal asymptote which is independent on the kind of polymers or the concentration of the solution, and is known in literature as the Maximum Drag Reduction asymptote (MDR).

Based on experimental data, Virk (1971a) proposed that an elastic buffer region exists between the viscous sublayer and the turbulent core, in which the velocity profile follows

$$u^+ = 11.7\ln y^+ - 17. \tag{1.5a}$$

L'vov et al. (2004) found that Eq. (1.5a) can be alternatively written by

$$u^+ = \frac{1}{K_v}\ln(eK_v y^+). \tag{1.5b}$$

where $1/K_\nu = 11.7$. Benzi et al. (2004) proposed that Eq. (1.5b) is an edge solution of the Navier–Stokes equation with an effective viscosity profile, and beyond which no turbulent solutions exist.

Virk's three layer model includes the viscous sublayer, a buffer or elastic layer and the turbulent core. Reischman and Tiederman (1975) using their data found that, in the elastic region, the velocity profile can be expressed as

$$u^+ = 7.687\ln y^+ - 8. \tag{1.6}$$

Virk's model has been widely adopted by researchers (Gasljevic et al., 2001; Larson, 2003; Min et al., 2003a; Min et al., 2003b). However, extensive research based on Virk's suggestion has not yet produced a definitive understanding of the mechanism of the velocity profile that combines the interaction of polymer and turbulent structures. Several experimental results (Gyr and Bewersdorff, 1995) show that Virk's assumption of the ultimate velocity profile may not be generally true, i.e., whether or not a specific additive has its own MDR asymptote is still an open question.

The third phenomenon of drag reducing flow is the existence of "shear deficit", which has been discovered and reported by many researchers, i.e. Willmarth et al. (1987); Gyr and Tsinober (1997); Toonder et al. (1997) and Warholic et al. (1999). From experimental observations, it is found that the total shear stress in drag reducing flow is greater than the sum of viscous shear stress ($\nu du/dy$) and the measured Reynolds shear stress ($-\overline{u'v'}$), the shear deficit can be expressed in the following form

$$G(y) = \frac{\tau}{\rho} - \left[\nu\frac{du}{dy} + (-\overline{u'v'})\right], \tag{1.7a}$$

where $G(y)$ is the shear deficit; τ is the total shear stress, and

$$\tau = \rho u_*^2\left(1 - \frac{y}{r}\right). \tag{1.7b}$$

Obviously, $G(y)$ is zero in a Newtonian fluid flow, but it is essentially non-negligible and mostly positive due to the elastic effect for a drag reducing flow with polymer additives (Gyr and Tsinober, 1997). $G(y)$ can be expressed as

$$G(y) = \nu_{eff}\frac{du}{dy}, \tag{1.8}$$

where ν_{eff} is the effective viscosity which was first introduced by Giesekus (1981). Eq. (1.8) is a simple and direct way for representing the non-Newtonian effects.

The objectives of this study are: (1) to theoretically investigate the mean velocity profile and friction factor of drag reducing flow in smooth pipe/channels; (2) to discuss the onset Reynolds number for drag reduction; and (3) to quantify the optimal polymer concentration for MDR.

2. Mean velocity profile in smooth pipe flows

Eq. (1.8) is a convenient and instructive way for representing quantitatively the viscoelastic effects as it states how much the viscosity would have to increase locally to balance the momentum deficit by a pseudo-viscous momentum exchange (Gyr and Tsinober, 1997). Using an analogy with Boussinesq's expression for the eddy viscosity in turbulence, it is postulated that the effective viscosity can be expressed by

$$\nu_{eff} = \alpha_* u_* r, \tag{2.1}$$

where α_* is an elastic factor.

Ptasinski et al. (2001) measured the Reynolds shear and viscous shear stresses in a drag-reducing pipe flow using a two-component LDA system. Their results are shown in Fig. 1, where the total shear stress is represented by the straight line; the open symbols denote the measured Reynolds shear stress $-\overline{u'v'}/u_*^2$ and measured viscous shear stress du^+/dy^+; the solid symbols are the $G(y)/u_*^2$ which is computed from Eq. (1.7a) and (1.7b) using the measured viscous and Reynolds shear stresses. Fig. 1 shows that the trend of the shear deficit $G(y)$ is similar to that of the viscous shear stress; this implies that the shear deficit is proportional to the velocity gradient, du^+/dy^+. Substituting Eq. (2.1) into Eq. (1.7a) and (1.7b), one obtains

$$u_*^2\left(1 - \frac{y}{r}\right) = (\nu + \nu_{eff})\frac{du}{dy} - \overline{u'v'} = \nu D_*\frac{du}{dy} - \overline{u'v'}, \tag{2.2}$$

where D_* is the drag reduction parameter and

$$D_* = 1 + \alpha_*\frac{u_* r}{\nu}. \tag{2.3}$$

The elasticity of polymer solution mainly depends on the molecular weight, concentration and intrinsic viscosity of polymer (Mun et al., 1998). Dou (1996) analyzed the elastic factor α_* and expressed it as (see Appendix)

$$\alpha_* = A[\eta]C\alpha_o\exp(-B\alpha_o^{0.7}C_w). \tag{2.4}$$

in which A is a constant and equal to $\pi^2/15$; B is an empirical factor and equal to 25500; $[\eta]$ is the intrinsic viscosity of polymer; C is the concentration of solution (g/cm^3); C_w is the concentration of

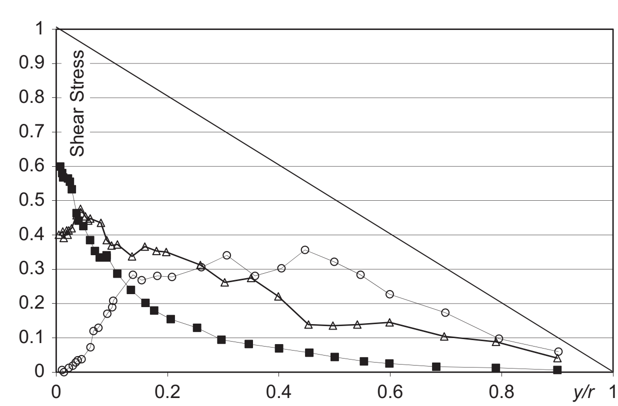

Fig. 1. Shear deficit in a drag-reducing pipe flow, after Ptasinski et al. (2001), in which the straight line represents the total shear stress $\tau/\rho u_*^2$; open circles denote measured Reynolds shear stress, i.e., $-\overline{u'v'}/u_*^2$; solid squares are the measured viscous shear stress, i.e., du^+/dy^+, and open triangles are the shear stress deficit, i.e., $G(y)/u_*^2$.