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A novel Lagrangian algebraic slip mixture model for two-phase flow in horizontal pipe

Zhi Shang*, Jing Lou, Hongying Li

*Institute of High Performance Computing, Agency for Science, Technology and Research (A*STAR), 1 Fusionopolis Way, #16-16 Connexis, Singapore 138632, Singapore*

HIGHLIGHTS

- A novel mathematical model was established for multiphase flow.
- This model realizes the connection between Eulerian model and Lagrangian model.
- Numerical simulations were performed for the mathematical models.
- Validations were carried out within wide regimes in a horizontal pipe.

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ABSTRACT

A novel Lagrangian algebraic slip mixture model (LASMM) has been developed to study multiphase flows. In this model, the slip velocity between continual and dispersed phases was developed through the bubble Lagrangian movement equation. According to the Lagrangian equation, the various interfacial forces at interface of the continual and dispersed phases were considered. Through the connection of the slip velocity, the Lagrangian equation was induced into the governing Eulerian equations of the two-phase mixture flow. This model therefore realized the connection between Eulerian model and Lagrangian model. Through the comparisons of the numerical simulations to the experiments in horizontal pipe, this model was validated.

1. Introduction

In fluid mechanics, two-phase flow includes a flow system containing gas and liquid flowing together. In engineering, gas–liquid two-phase flow can be encountered in many industrial processes such as chemical reactors, steam generators for nuclear power plants and petrochemical pipe flows. In the practical two-phase flows, for example in the oil industry, the gas-lift is often applied to decrease the hydrostatic weight of the pipe flow (Guet et al., 2004). In gas–liquid two-phase flow system, gas phase is usually dispersed into liquid in the form of bubbles. Such two-phase flow is also widely encountered in horizontal pipe flows.

In horizontal gas–liquid two-phase flows, the deformations of bubbles as well as their interactions each other have significant effects on the flow hydrodynamics. The interactions among multiple bubbles include coalescence or breakup. Due to the buoyancy, the dispersed bubbles tend to migrate towards the top of the pipe. Therefore a highly non-symmetric volume distribution will generate in the pipe cross-section (Kocamustafaogullari and Wang, 1991; Kocamustafaogullari and Huang, 1994; Yuan et al., 2011; Ekambara et al., 2012).

The experiment methods usually were employed to explore the complex internal flow structure of horizontal two-phase flows. Krokovni et al. (1973) used electrodiffusion method to measure the friction of horizontal two-phase flow. Kocamustafaogullari and Wang (1991) and Kocamustafaogullari and Huang (1994) used double-sensor electrical resistivity probe to measure the void fraction, interfacial area concentration, bubble size and velocities in a horizontal pipe with the test section of 50.3 mm inner diameter and 15.4 m in length. Vallee et al. (2008) used high-speed video camera with a particle image velocimetry (PIV) system to observe the flow structure and used piezoelectric transducers to measure the pressure drop in a 2 m long horizontal duct with the rectangular cross

* Corresponding author. Tel.: +65 64191547.
E-mail addresses: shangzhi@tsinghua.org.cn, shangz@ihpc.a-star.edu.sg (Z. Shang).

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section of 250 mm × 50 mm. From experiments, the flow structure and specific physical parameters can be measured directly. However, normally the cost of doing experiments is very high. Nowadays following the advantage of computer technology, the flow structure and specific physical parameters in horizontal two-phase flows can be simulated numerically based on computational fluid dynamics (CFD) methodology (Vallee et al., 2008; Yuan et al., 2011; Ekambara et al., 2012).

Usually the accuracy of CFD will depend on the models, for example, homogeneous, heterogeneous and two-fluid models, which are able to perform the numerical simulations for two-phase flows (Sanyal et al., 1999; Lain et al., 2002; Yuan et al., 2011). The algebraic slip mixture model (ASMM) is often employed to simulate gas–liquid two-phase flows. In traditional ASMM, the slip velocity between different phases is simply presented by the acceleration generated by the centrifugal force (Sanyal et al., 1999; Brennan, 2001; Chen et al., 2005). However, the interfacial forces, such as drag force, lift force, virtual mass force, wall lubrication force, turbulent dispersion force etc., are not included. Hence the accuracy of the traditional ASMM will be affected.

A novel Lagrangian algebraic slip mixture model (LASMM) was developed in this paper. It employed a mixture model to describe the two-phase flows based on Eulerian model. The slip velocity, which can be developed from the dynamic equation of the dispersed phase based on Lagrangian model, was introduced to represent the difference between dispersed and continuous phases. Owing to the Lagrangian model, the interfacial forces, such as buoyancy, drag force, lift force, virtual mass force, wall lubrication force, turbulent dispersion force etc., are able to be involved. It is therefore different from the traditional ASMM provided by FLUENT, CFX and other commercial software. Through comparisons to experiments and two-fluid model on horizontal two-phase flows, this model was validated.

2. Mathematical model

Considering a problem of turbulent multi-component multi-phase flow with one continuous phase and several dispersed phases, the time average conservation equations of mass, momentum and energy for the LASMM as well as the turbulent kinetic energy equation and the turbulent kinetic energy transport equation can be written as the following:

$$\partial\rho_m/\partial t+\nabla\cdot(\rho_m U_m)=0 \tag{1}$$

$$\partial(\rho_m U_m)/\partial t+\nabla\cdot(\rho_m U_m U_m)=-\nabla p+\rho_m g+\nabla\cdot[(\mu_m+\mu_t)(\nabla U_m+\nabla U_m^T)] -\nabla\cdot\sum\alpha_k\rho_k U_{km}U_{km} \tag{2}$$

$$\partial(\rho_m h_m)/\partial t+\nabla\cdot(\rho_m U_m h_m)=q+\nabla\cdot\left[\left(\frac{\mu_m}{Pr}+\frac{\mu_t}{Pr_t}\right)\nabla h_m\right]-\nabla\cdot\sum\alpha_k\rho_k h_k U_{km} \tag{3}$$

$$\partial(\rho_m k)/\partial t+\nabla\cdot(\rho_m U_m k)=\nabla\cdot\left[\left(\mu_m+\frac{\mu_t}{\sigma_k}\right)\nabla k\right]+G-\rho_m\varepsilon \tag{4}$$

$$\partial(\rho_m \varepsilon)/\partial t+\nabla\cdot(\rho_m U_m \varepsilon)=\nabla\cdot\left[\left(\mu_m+\frac{\mu_t}{\sigma_\varepsilon}\right)\nabla \varepsilon\right]+\frac{\varepsilon}{k}(C_1 G-C_2\rho_m\varepsilon) \tag{5}$$

in which

$$\rho_m=\sum\alpha_k\rho_k \tag{6}$$

$$\mu_m=\sum\alpha_k\mu_k \tag{7}$$

$$\rho_m U_m=\sum\alpha_k\rho_k U_k \tag{8}$$

$$U_{km}=U_k-U_m \tag{9}$$

$$G=\tfrac{1}{2}\mu_t[\nabla U_m+(\nabla U_m)^T]^2 \tag{10}$$

$$\mu_t=C_\mu\rho_m\frac{k^2}{\varepsilon} \tag{11}$$

where ρ is the density, U is the velocity vector, α is the volumetric fraction, p is pressure, g is the gravitational acceleration vector, U_{km} is the diffusion velocity vector of k dispersed phase relative to the averaged mixture flow, h is enthalpy, q is heat input, μ is viscosity, μ_t is turbulent viscosity, Pr is molecular Prandtl number, Pr_t is turbulent Prandtl number, G is stress production. C_μ, σ_k, σ_ε, C_1, C_2 are constants for standard k–ε turbulence model (Launder and Spalding, 1974), shown in Table 1. The subscript m stands for the averaged mixture flow, and k stands for k dispersed phase.

In addition to the above equations, the following conservation equation for each phase is also necessary:

$$\partial(\alpha_k\rho_k)/\partial t+\nabla\cdot(\alpha_k\rho_k U_m)=\Gamma_k-\nabla\cdot(\alpha_k\rho_k U_{km}) \tag{12}$$

where Γ_k is the generation rate of k-phase.

In order to closure the governing equations (1)–(12), it is necessary to determine the diffusion velocities U_{km}. The following equation is employed to convert the diffusion velocities to slip velocities that can be defined as $U_{kl}=U_k-U_l$.

$$U_{km}=U_{kl}-\sum\frac{\alpha_k\rho_k}{\rho_m}U_{kl} \tag{13}$$

Actually the above equation can be developed from the definition of the mixture density equation (6), the definition of mixture mass flux equation (8), the diffusion velocity equation (9) and the slip velocity U_{kl}. Once the slip velocities are obtained, the whole governing equations will be closured.

Because the slip velocities present the difference of the movement between the dispersed phase for instance gas and the continuous phase for instance liquid. The dispersed phase can be presented by its own law of motion. For example, in gas and liquid two-phase flow system, the following equation can be use to describe the single bubble movement inside a liquid, which normally is called as Lagragian equation of motion.

$$F_g=F_{bouyancy}+F_{drag}+F_{virtual}+F_{lift}+F_{\text{lubrication}}+F_{dispersion}+\cdots \tag{14}$$

where F_g is the inertia force acting on the bubble due to its acceleration, $F_{bouyancy}$ is the force due to gravity and buoyancy, F_{drag} is force due to drag by the continuous liquid, $F_{virtual}$ is the force due to virtual mass effect, F_{lift} is the force due to transverse lift, $F_{lubrication}$ is the wall lubrication force caused by the liquid flow rate which between gas bubble and the solid wall is lower than that between the gas bubble and the main flow, $F_{dispersion}$ is the turbulent dispersion force due to the movement of the turbulent eddies, and so on the other forces can be added into Eq. (14).

In this paper, only the forces of buoyancy, drag, virtual mass, transverse lift, wall lubrication and turbulent dispersion were considered. The expanded description about these forces can be represented as the following equations (Lain et al., 2002; Zhou et al., 2002; Yeoh and Tu, 2006).

$$F_g=\rho_g\frac{dU_g}{dt} \tag{15}$$

$$F_{bouyancy}=(\rho_g-\rho_l)g \tag{16}$$

Table 1
Constants of standard k–ε turbulence model.

Variable	C_μ	σ_k	σ_ε	C_1	C_2
Constant	0.09	1.0	1.3	1.44	1.92