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Using the breakage matrix approach to define the optimal particle size distribution of the input material in a milling operation

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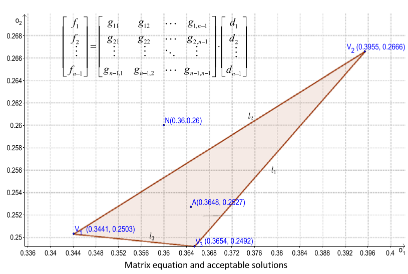
HIGHLIGHTS

- A procedure for solving the reverse problem of the breakage matrix is proposed.
- Mathematical solution and perceived limitations of the approach were discussed.
- The dimension of the breakage matrix determines the procedure.
- Applicability of the proposed model was confirmed by some examples.

GRAPHICAL ABSTRACT



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ABSTRACT

The breakage matrix approach proved to be a useful tool in describing the milling processes. Having determined the breakage matrix for any particle size distribution (PSD) of input material, the PSD of output can be predicted. In this paper, the reversibility of the aforementioned approach is examined. Perceived limitations of the breakage matrix approach are considered within the mathematical calculation. It is established that the procedure is determined by the dimension of the breakage matrix (relation between the number of input and output size fractions). The paper shows that the breakage matrix approach can be used to define the PSD of the input material to a milling operation which would give the desired PSD of the output material.

1. Introduction

Comminution is an important unit operation in many industries that process food, ceramics, chemicals, pharmaceuticals etc. (Prasher, 1987). Particle size distribution (PSD) of solids is often an important quality factor while various types of size reducing equipment are used. Particle degradation can be an intentional or undesirable side effect of normal transport and handling operations. Size reduction is achieved by mechanical forces (compression, impact, and shear) that cause rupture. Usually in any given machine, one of the forces is more dominant and important than the others (Fellows, 1988).

Population balance type of modeling is most often used to mathematically describe the comminution process (Epstein, 1948; Austin, 1971; Austin and Bagga, 1981; Meloy and Williams, 1992; Verkoeijen et al., 2002; Bilgili and Scarlett, 2005; Hennart et al., 2009). This model is based on calculating time-dependent rates of breakage. Practically, this means the evolution of PSDs over time. The main problem with such an approach is that it requires continual intra-process sampling. This is often difficult or sometimes even impossible. A possible alternative is to treat an entire process as a single breakage event and to find an overall relationship between input and output PSDs (Baxter et al., 2004).

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The so-called breakage matrix approach can be successfully used for this purpose. Although this approach is applicable to practical experiments, it is less informative than population balance type approaches. Normally, it is used to predict the PSD of the output material from a milling operation. In this paper, the breakage matrix approach is used to define the PSD of input material to a milling operation which would give the desired PSD of output (reverse problem). This could be useful in milling operations where the possibility of controlling PSD of the input material exists. According to our knowledge, the reverse problem is not methodically studied in the breakage matrix context. However, similar problem is studied in determination of bubble size distribution in food (Campbell et al., 1999), where input and output PSD correspond to distributions of circle and sphere sizes. In Campbell et al. (1999) it is proposed that the mentioned problem could be solved by singular value decomposition (SVD) method (Press et al., 1992). Especially, in the case of overdetermined set of linear equations, SVD method produces a solution that is the best approximation in the least-squares sense.

2. The breakage matrix approach

The idea of using breakage matrices to relate input and output PSDs was first introduced by Broadbent and Callcott (1956a, 1956b, 1957) in the form of a matrix equation:

$$B \cdot f = o \tag{1}$$

where f and o are vectors describing the input and output PSDs respectively. B is the breakage matrix relating f and o by the rules of matrix multiplication. Vectors f and o could be the weight fractions on each sieve from a sieve analysis, making this approach applicable to practical experiments. Elements of the breakage matrix B can be determined by milling mono-sized or narrow sized range fractions of inlet material. The PSD from each mono (narrow) sized fraction forms the corresponding column of the breakage matrix. Coefficients b_{ij} represent the weight fraction of the size range j of the output material obtained by milling the size range i of the input material to a milling operation. Having determined the breakage matrix for any particle size distribution of input material, particle size distribution of output can be predicted. In expanded form, Eq. (1) is rewritten as

$$\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ b_{m1} & b_{m2} & \vdots & b_{mn} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix} = \begin{bmatrix} o_1 \\ o_2 \\ \vdots \\ o_m \end{bmatrix} \tag{2}$$

where

$$\sum_{j=1}^{n} f_j = 1, \tag{3}$$

$$\sum_{i=1}^{m} o_i = 1, \tag{4}$$

$$\sum_{i=1}^{m} b_{ij} = 1 \quad \text{for each } j \in [1, \ldots, n]. \tag{5}$$

The breakage matrix is effectively a combination of the selection function $S(x)$ and the breakage function $B(y,x)$. The selection function is a mass fraction of particles that are selected and broken in time (during the process). The breakage function is a mass fraction of breakage products from size x that fall below size y, where $x \geq y$ (Holdich, 2002). The selection function is encompassed in diagonal elements of the breakage matrix, and the breakage function is in the off-diagonal elements (Baxter et al., 2004).

Broadbent and Callcott (1956a, 1956b, 1957) used the same sieve sizes for both feed and product size distributions, giving a square breakage matrix, where the above diagonal values are equal to zero. Usually, the PSDs of the feed and milling output occur in different size ranges. Applicability of the breakage matrix approach for predicting the PSD on the first break milling of wheat was confirmed by Campbell and Webb (2001) and Campbell et al. (2001). Using different sized sieves (and number of sieves) for inlet and outlet size distributions, the breakage matrices of Campbell and Webb were non-square and therefore more general and accurate than that of Broadbent and Callcot. The breakage matrix approach has also been successfully used to predict compositional distribution of broken particles along with their size distribution (Fistes and Tanovic, 2006).

Limitation of the breakage matrix approach is that the breakage matrix determined for one set of conditions cannot be adopted for another set of conditions (Campbell et al., 2001). The approach also presumes that particle breakage is independent of any inter-particle interactions; the particles of the same size, milled on the same set of conditions, break the same way regardless of whether they belong to a mono-dispersed sample or a poly-dispersed mixture (Campbell and Webb, 2001). However, some recent papers deal with the modified breakage matrix methodology in order to characterize multi-particle interactions during breakage (Baxter et al., 2004; Bilgili and Capece, 2011, 2012; Capece et al., 2011).

Considering Eq. (1) the output PSD (vector o) can be altered by changing the breakage matrix (B) or by changing the input PSD (vector f). During comminution operations, both material properties and milling methods affect particle breakage (Scanlon and Lamb, 1995). The factors affecting particle size reduction can be classified into those arising from the pshysico-chemical properties of the material and those related to the design and operation of the milling equipment (Campbell et al., 2001). Sometimes, the most efficient way to influence the outlet PSD is by changing the process parameters (higher rotation speed of a tumbling mill, gap of a roll mill etc.). In such way the desired outlet PSD is obtained by changing the breakage matrix (B). The significance of the forward problem (finding the outlet PSD) and the inverse problem (parameter identification) is well-established in literature. The other possible approach, which has been addressed in this paper, is to identify the input PSD that would result in a desired output PSD. This approach implies that the breakage matrix (B) is constant and therefore it applies to the same set of milling conditions.

In this paper it is assumed that the particle breakage is independent of any inter-particle interactions. Also, it is presumed that the PSDs of the input and output occur in different size ranges. Moreover, the study is carried out as a general case where a number of inputs and outputs could be different and the elements of the breakage matrix are arbitrary values between 0 and 1.

The paper is organized as follows. Section 3 gives the mathematical procedure for calculating the PSD of input material to a milling operation which would give the desired PSD of the output. This is done by transformation of the matrix equation (2) using standard Gaussian elimination techniques. Also, in case $m > n$ SVD method is used to obtain the best-fit solution. In comparison with direct calculation of Eq. (2), given procedure is more complicated. Namely, in direct way conditions (3) and (5) imply that the output value is always in the interval defined by maximum and minimum in the appropriate row of the breakage matrix. In the opposite direction it is possible to get input values greater than 1 and as a consequence some of the input values are negative. To avoid that, the problem is observed as a system of linear inequalities and it is solved by a linear programming method. This is done in Section 4. Moreover, a geometrical approach is applicable when dimension of the breakage matrix is $n \times 3$. The proposed model is tested by examples. The data used in the examples relate to wheat milling.