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On the uncertainty quantification of blood flow viscosity models

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HIGHLIGHTS

- A 3D Navier–Stokes model for blood flow was considered under uncertainty.
- Three different approaches to the blood model uncertainty were considered.
- A Polynomial Chaos Non-Intrusive method was applied to viscosity uncertainty.
- Wall shear force and pressure was quantified with viscosity as random variable.
- Uncertainty quantification of the random variables on the solutions.

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ABSTRACT

The blood viscosity uncertainty is investigated in an idealized portal-vein flow and its effect is propagated in the 3D Navier–Stokes equations and quantified on the quantities of interest, such as wall shear, pressure and mass flow split. The variability of the random blood viscosity is investigated in detail assuming that the true blood viscosity is given in the range covered by four Carreau blood viscosity model variants. Three different characterizations of the associated Probability Density Functions (PDFs) were considered: (i) a single blood Carreau model with random parameters that covers the variability range under consideration; (ii) the assumption that there is equal probability of sampling each of the four different Carreau model variants; and (iii) the assumption of a bi-linear composition of the four Carreau models affected by random coefficients. These assumptions result in different inlet blood viscosity PDFs that were propagated in the Navier–Stokes solution with the application of a non-intrusive stochastic collocation method based on the generalized polynomial chaos expansion. The stochastic simulations have quantified the uncertainty of random viscosity model parameters on the interested flow parameters wall shear stress and pressure for two Reynolds numbers: Re=212 and Re=21. The results include error bars of these variables and hierarchy impact of the random variables on the solution.

1. Introduction

Many fields of engineering are increasingly aware of uncertainty quantification by which various uncertainties can be quantified and understood. Hemodynamics simulation studies have been frequently used to gain a better understanding of functional diagnostic and therapeutic aspects of blood flow (Ho et al., 2010). They will be more trusty accepted if the simulation is robust against uncertainties. Portal vein thrombosis and clot disturbance of hepatic blood flow are diseases that may induce hemodynamic perturbations (Congly and Lee, 2013). This configuration is less studied than, for example, the aorta blood flow dynamics and is very often concerned with reduced blood flow rate due to liver diseases, such as cirrhosis portal vein hypertension evolution or thrombosis (George, 2008).

Hemodynamics simulation studies have been frequently used to gain a better understanding of functional diagnostic and therapeutic aspects of blood flow (Ho et al., 2010). They will be more trusty accepted if the simulation is robust against uncertainties. The present work deals with the problem of uncertainty in blood viscosity and the 3D portal vein was selected. In the absence of MRI data an idealized geometry was considered (Petkova et al., 2003; George, 2008) for its 3D reconstruction. Several geometrical models with and without clot have been proposed (Petkova, 2008; Botar et al., 2010; Ho et al., 2012). The presence of clot narrows the vessel geometry and creates a region of high and low shear stresses. The considered idealized geometry branches twice and the flow symmetry is broken by a thrombosis clot after the first bifurcation and also by different outlet pressure boundary conditions.

In addition, there is a need to eliminate the confusion between uncertainty sources that yield flow parameters modifications due to

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blood model parameters or geometry variability in physiological conditions. The present methodology is applied only to blood viscosity uncertainty but can be applied to any other flow geometrical configuration.

Blood flow simulations of patient-specific vessel geometries face the problem of uncertainty from several parameters (a detailed description is given by Sankaran and Marsden (2011)): vessel geometry from image reconstruction, boundary conditions, material properties, blood viscosity model, etc.

Although there has been a considerable amount of viscosity models in blood rheology, none of them has been commonly agreed upon. None of the models fully expresses the effects of the extremely complicated nature of blood rheology and its dependence on many factors (Johnston et al., 2006). The blood properties are dependent on cell concentration, coagulation, adhesion and oxygen concentration. The viscosity of blood varies with the hematocrit, which is the percentage of the total blood volume occupied by blood cells. Recent multiscale calculations have attempted to include its effect (Nakamura and Wada, 2010) and the implications of important characteristics of blood microstructure at low shear rates, such as the time and flow dependent characteristics of the red blood cell network have been theoretically investigated by Kaliviotis and Yianneskis (2009, 2011). Different blood viscosity models are used with various fitting parameters to take into account the different hematocrit values and to obviate the need to acquire patient-specific rheological data. Blood has a shear-thinning behavior but the Non-Newtonian fluid is only remarkably important at low shear rates and also in small arteries and capillaries with strain rates lower than 100 s^{-1}. In large arteries the blood behaves as a Newtonian fluid. The mixed behavior is still not totally agreed upon and there are several models to simulate this behavior as, for example, the power-law, the Casson model and the Carreau–Yasuda models, with no certainty in which one best represents the behavior of the blood viscosity. Consequently, there are problems not only about the selection of the viscosity model for deterministic simulations but also for stochastic calculations where the characterization of the random function (viscosity model) is required. The present work is focused on the general problem of the characterization of the input random variables and particularly on blood viscosity uncertainty.

The quantification of these uncertainties on the flow solution and particularly on the quantities of interest (as, for instance, wall shear stresses, pressure, etc.) requires three main assumptions: (i) the selection of the stochastic model to solve the Navier–Stokes equations with random variables; (ii) the statistical characterization of the random variables—mean, variability and Probability Density Function (PDF); and (iii) the methodology to investigate the propagation of the uncertainties to allow their ranking on the solution.

The first issue refers to the parametric uncertainty and its quantification is typically studied with Monte Carlo techniques, which essentially perform a statistical analysis on the solutions of deterministic simulations with randomly selected conditions. This approach is non-intrusive, in the sense that it is not necessary to change the deterministic fluid flow solver, but it does not readily provide information about the sensitivity of the model output to specific parametric uncertainty. In its most fundamental form, sampling does not retain the functional dependence of output on input but rather produces quantities that have been averaged simultaneously over all input parameters. In addition, it has a very high computational cost, mostly when the model is computationally demanding, and often precludes the possibility of carrying out a large ensemble of model runs. Furthermore, the understanding of low-probability high-consequence events is difficult using standard Monte Carlo schemes because such events are rarely generated. Recently, several alternative approaches to Monte Carlo techniques that are based on the Polynomial Chaos (PC) expansion are becoming popular (Wiener, 1938; Ghanem and Spanos, 1991). Later development of chaos decompositions based on non-Gaussian basic variables was made in Xiu and Karniadakis (2002a) which has been called the generalized Polynomial Chaos (gPC) and uses the orthogonal polynomials from the Askey family with weighting functions similar to probability functions (Beta, Gama, binomial, etc). Local PC expansions, suited for long-term integration and discontinuities in stochastic differential equations, were studied in Le Maître et al. (2001) and Wan and Karniadakis (2005). The Intrusive Spectral Projection (ISP) methods have the advantage of being very efficient on solving problems with stochastic variables but require a reformulation of the deterministic governing equations for a particular problem (Sereno et al., 2010), which sometimes is not practical or even impossible in the case of existing complex or commercial codes. For these cases there is a non-intrusive alternative called Non-Intrusive Spectral Projection (NISP) in which simulations are performed at specific collocation points in the stochastic space (Xiu and Hesthaven, 2005). The NISP technique has the advantage, compared with Monte Carlo techniques, of providing direct sensitivity information about the propagation of parameters uncertainty through a system (Xiu and Hesthaven, 2005) and the sampling process can be optimized. For low number of variables one can easily deal with the NISP approach of the PC expansion because it uses a numerical projection into the stochastic space instead of the exact (or quasi) projection taken by the intrusive methodology. NISP techniques are becoming very attractive for application in scientific research and engineering practice (Reagan et al., 2003; Babuskă et al., 2004; Xiu, 2007; Ganapathysubramanian and Zabaras, 2007; Mendes et al., 2011). The NISP propagates the full probabilistic representation of the model inputs into the model outputs and allows to evaluate dependencies related to the specific uncertainty in a given parameter, while sensitivity analysis compares equal parametric perturbations. Few applications of NISP techniques to stochastic cardiovascular flows have been reported. A systematical study of parametric data uncertainty in 1D model of human arterial networks was performed by Xiu and Sherwin (2007) and a flexible wall 1D simulations have been performed by Grinberg et al. (2011) to the complete circle of Willis model to account for the uncertainty in the elasticity of the arterial wall parameters on flow rates and mean pressure. Another detailed study was performed by Sankaran and Marsden (2011) about the confidence in the output of hemodynamic simulations based in Navier–Stokes solutions of the input uncertainties including variations in boundary inflow velocities, geometry and outflow resistances and impedances on several cardiovascular problems.

The second issue is related with the characterization of the uncertainty and to the Authors knowledge it was not considered before.

For the present case it is assumed that the blood viscosity general behavior is given by the Carreau family of models, which has many different versions depending on the four model parameters considered. We propose three methodologies to built up the PDF for the random variable characterization:

(i) model parameters uncertainty; The model corresponds to the common parametric uncertainty practice where the random blood viscosity is characterized by a mean and a variability with a prescribed PDF. The uncertainty is given by the non-linear square fit method used in Carreau blood viscosity model.
(ii) model uncertainty from a mixture of models, each one with a probability of occurrence; The model corresponds to a different way to consider the blood viscosity uncertainty. Here is assumed that there is equal probability to select one of the four different blood viscosity models of Carreau type. This