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Alternate rotating walls for thermal chaotic mixing

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ABSTRACT

In this study, we numerically investigate the evolution of two-dimensional mixing and heat transfer enhancement within a two-rod stirring device. The fluid is heated by the walls, which are maintained at a constant temperature. We show by analysis of different stirring protocols that the use of discontinuous wall rotations is necessary to promote heat transfer by chaotic mixing. This condition is also required to avoid hot spots in the vicinity of the walls. The statistics of temperature scalars (mean and standard deviation of dimensionless temperature fields) allow us to determine the influence of geometrical and physical parameters on mixing and heating performance. Thermal strange eigenmodes are revealed during the mixing process by the development of complex recurrent patterns, and the self-similar character of temperature evolutions is confirmed by the probability distribution functions of the rescaled non-dimensional temperature.

1. Introduction

The topic of heat transfer and mixing plays an important role in many fields as various as earth and life sciences, chemical engineering and material science. This subject is particularly difficult to solve when highly viscous fluids are concerned. A variety of industrial processes involve the heating of highly viscous liquids; they include material processing (molten polymers for example), petroleum engineering, food engineering or environmental engineering. The motivation for this study is to analyze the heating and mixing performance of a chaotic mixer with a rather simple design. Chaotic mixing is obtained by considering the chaotic advection phenomenon [1–3] that occurs in laminar flow. Today it is well recognized that chaotic mixing is the most efficient mixing process when highly viscous fluids are involved [4]. Chaotic mixing is also recommended for the mixing of delicate fluids that do not resist the high strains encountered in turbulent flows. Despite the simplicity of their velocity fields, chaotic advection flows are able to create very complex patterns of the advected scalar with highly stretched and folded structures. The more numerous are very thin striations produced by the flow, the more efficient the diffusion and the faster the homogenization of the scalar. When heat transfer is considered, temperature is the scalar and heat conduction is the diffusion source. In this study, we focus only on the relationship between chaotic mixing and the enhancement of heat transfer in a particular flow. Despite its obvious industrial relevance, there are only a limited number of works that consider this problem. In fact, many more works in the literature concern the relationship between chaotic mixing and chemical reaction more than the coupling of chaotic mixing with heat transfer. When heat transfer enhancement is involved, mainly two classes of flow geometries are encountered in the production of chaotic mixing: those that use rotating elements as eccentric cylinders [5–8] and those that use multiple pipe bends [9–19]. For the latter geometries, chaotic trajectories are determined by continuous modification of pipe wall design; the third space dimension provides the required additional degree of freedom and no displacement of the wall is imposed. As a consequence, the number of operating parameters that determines the options for effective stirring protocols is reduced, and the flow structure is given only by the chosen pressure gradient (i.e., the Reynolds number for a particular fluid). Another interesting 3D geometry is the continuous chaotic flow generated inside the Rotated Arc Mixer of Lester et al. [20]. The mixer consists of an inner stationary cylinder lacking internal structure, with apertures cut through its wall, wrapped by a concentric outer rotating cylinder. This geometry offers a large, tunable, optimization parameter space for the enhancement of heat and/or mass transfer. For all of the cases discussed above, thermal chaotic mixing is used to attain multiple objectives: to encourage heat transfer between the heated walls and the neighboring fluid and to achieve temperature homogeneity for the whole fluid domain. For this last condition, we need to both prevent the

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Nomenclature

A_c	surface of a mesh element (m^2)
$A_{T\sigma}$	composite mixing indicator
c_p	heat capacity ($J\,kg^{-1}\,K^{-1}$)
d	vector joining the centers of two cells
E	energy (J)
k	thermal conductivity ($W\,m^{-1}\,K^{-1}$)
$\vec{n}$	normal oriented vector
p	pressure (Pa)
R_3	tank radius (m)
R_1, R_2	rod radii (m)
s	strip width (m)
S_{tot}	total surface of the fluid (m^2)
t	time (s)
T	temperature (K)
U	tangential velocity
$\vec{U}$	velocity field ($m\,s^{-1}$)
V	volume (m^3)

Dimensionless numbers

Pe	Péclet number
Pr	Prandtl number
Re	Reynolds number
S_t	Strouhal number

T^*	dimensionless temperature
X	rescaled dimensionless temperature

Greek symbols

α	thermal diffusivity ($m^2\,s^{-1}$)
ε	eccentricity (m)
ϕ	generic scalar variable
Γ	generic diffusion coefficient
λ	stretching rate (s^{-1})
ρ	fluid density ($kg\,m^{-3}$)
σ	standard deviation
τ	period of modulation (s)
$\bar{\bar{\tau}}$	viscous stress tensor
Ω	angular velocity ($rad\,s^{-1}$)
χ_g	scalar temperature dissipation indicator

Subscripts

c	cell
m	mean
f	face of a cell

Superscript

$*$	dimensionless

formation of a large, unmixed KAM island, where the fluid remains cold, and also avoid hot spots in the vicinity of heated walls, which can degrade the fluid.

One of the main differences between thermal chaotic mixing and reactive chaotic mixing is due to the large contrast in transfer rates between mass and heat. Effectively, it is common to notice a difference of two decades or more between the values of the molecular and thermal diffusivities. Thermal diffusivities are higher than molecular diffusivities, which has consequences on the solutions obtained for the advection–diffusion problem. Typically, the patterns of the *strange eigenmodes* [21,22] obtained for the scalar fields will differ. These patterns appear repeatedly for periodic velocity fields and repeat themselves every period with an exponential decay of the scalar contrast. More often, asymptotic transport is controlled by the slowest eigenmodes, but that is not always the case [23,24].

Thus, the main issue of this work is to analyze the heating and mixing performances in a realistic mixer of simple geometry and, in particular, to study the effect of wall rotation on the enhancement of heat transport in the whole fluid domain. This crucial effect of wall on the mixing efficiency has been recently highlighted by Gouillart et al. [25,26] for the homogenization of concentration in a 2D closed flow environment.

This article is organized as follows: in Section 2, we present the problem, the geometrical description of the mixer, the fluid properties and the flow parameters. We also describe the different types of stirring protocols used to study the performance of the mixer for heat transport. The section ends with a presentation of the governing equations. In Section 3, the numerical methods used to perform the simulations are described; in Section 4, the mixing and energy indicators defined to quantify the efficiency of heat transport and homogenization are presented. All the results are detailed and discussed in Section 5. At first, different flow topologies resulting from the choice of the stirring protocol imposed on the walls are shown, and their incidences on mixing and heat transport are given. The results are also analyzed with the help of temperature probability distribution functions (PDFs), which are very useful in globally characterizing the homogenization of temperature in time. We show the existence of recurrent patterns of the temperature fields and explain them in terms of thermal strange eigenmodes. We also analyze the temporal evolution of the temperature gradients in the fluid domain. We conclude the section with a study of the effect of rod eccentricity. Finally, in Section 6, concluding remarks are drawn and some perspectives for future work are given.

2. Problem statement

2.1. Geometrical description: the two-rod rotating mixer

A sketch of the studied mixer is presented in Fig. 1. It is composed of two circular rods of equal radii, which are maintained vertically inside a cylindrical tank (a bounded domain). The tank and the rods are heated and can rotate around their respective revolution axis. This geometry is similar to the two-roll-mills studied in

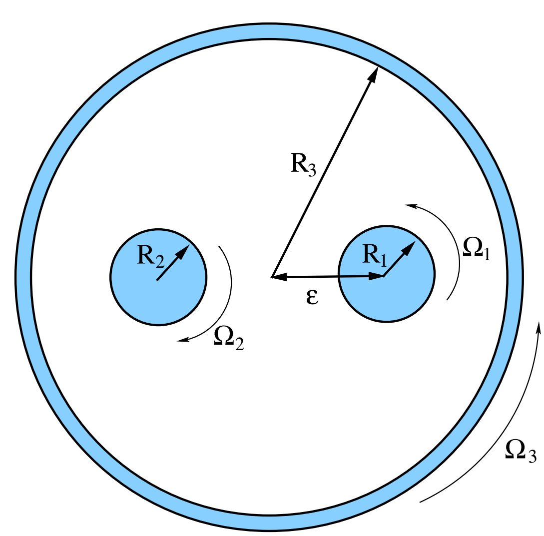

Fig. 1. Sketch of the two-rod mixer.